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Optimized planar Penning traps for quantum information studies

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Abstract Single electron qubits are attractive for quantum information processing because they offer, for example, the possibility of extremely long coherence times. For scaling up to a large number of coupled qubits, an array of planar Penning traps is a much more promising option than the cylindrical Penning traps within which one-quantum transitions have been observed. This report summarizes optimized trap configurations, discussed at length in Goldman and Gabrielse (Phys Rev A 81:052335, 2010), which promise to make it possible to realize one-electron qubits in a scalable configuration for the first time.

Keywords Quantum computing · Penning trap · Electron trapping

1 Introduction

A single trapped electron is an attractive qubit candidate since it is a two-level system with the possibility of a very long coherence time. High-fidelity detection of the quantum state has already been demonstrated via observation of quantum jumps [2] between the lowest cyclotron and spin states of an electron suspended in the magnetic field of a cylindrical Penning trap (Fig. 1). These observations made possible the most precise measurements of the electron magnetic moment and the fine-structure constant [3] and stimulated numerous proposals for realizing one-electron qubits [4–10]. For scaling to many one-electron qubits, however, traps constructed with their electrodes in a plane [11] are a more natural choice than cylindrical Penning traps. An essential first step is to implement in planar Penning

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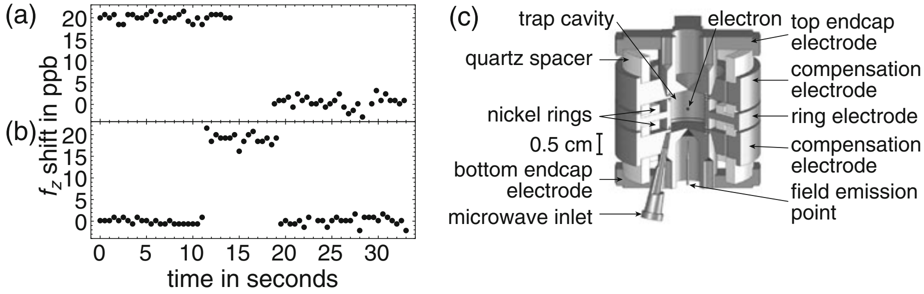


Fig. 1 **a** QND observation of a spin flip of one trapped electron. **b** QND observation of a one-quantum cyclotron transition for one electron. **c** Cylindrical Penning trap within which the electron is suspended

traps the methods for observing single electrons and single-quantum transitions that have been used so successfully in cylindrical Penning traps.

To date, the signal from many electrons simultaneously trapped within a single planar Penning trap has been observed [12–14]. However, a recent experimental report [14] concluded that it was "impossible" to detect a single electron within such a planar Penning trap because the "lack of mirror symmetry" makes it "impossible to create a genuinely harmonic potential" (unless perhaps traps that are smaller and more harmonic could be constructed [15]).

A long experience in designing traps for some of the most precise measurements in physics made us suspect that more optimistic conclusions might be possible if planar trap designs were carefully optimized for making one-electron qubits. The mentioned observations of one-quantum transitions and the accurate measurements [3] were enabled by a cylindrical Penning trap (Fig. 1c, [16]) designed so its electrodes form a microwave cavity that inhibits spontaneous emission. The most precise mass spectroscopy (e.g., [17]) was carried out in an orthogonalized hyperbolic trap [18, 19] designed to allow trapping potentials to be optimized without changing the trap depth. The most accurate comparison of q/m for an antiproton and proton [20], the most accurate one-ion measurements of bound electron g values [21, 22], and the most precise proton-to-electron mass ratio [23] were carried out in open-access Penning traps [24] designed to facilitate the introduction and transfer of trapped particles.

The results of our study were designs for optimized planar Penning traps [1], summarized here, reopening the possibility of one-electron qubits. The designs presented here greatly suppress the anharmonic part of the trapping potential; despite the asymmetry of the trap electrodes about the potential minimum, these trap configurations promise to realize potentials that are as harmonic as those achieved in cylindrical [16, 25, 26] traps and to produce amplitude-dependent frequency shifts that are orders of magnitude smaller than for previous planar trap designs.

Variations on planar Penning trap designs that are proposed offer methods for parallel detection or loading into a symmetric trap. These optimized planar Penning traps promise to make it possible to detect a single trapped electron in a planar Penning trap for the first time—an important first step toward a one-electron qubit.

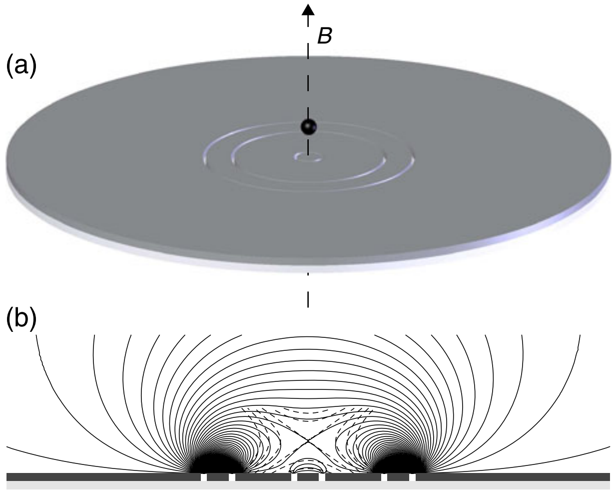


Fig. 2 **a** Three-gap planar trap with a trapped particle suspended above an electrode plane that extends to infinity. **b** Side view of trap electrodes and equipotentials spaced by V_0, with the infinitesimal gaps between the electrodes widened to make them visible. The equipotentials extend into the gaps between electrodes. The dashed equipotentials of an ideal quadrupole are superimposed near the trap center

2 Anharmonic axial oscillations

The crucial observable for realizing a one-electron qubit is the frequency of the axial oscillation of a trapped electron. One trapped particle will be observed in a planar Penning trap only if the oscillation frequency is well enough defined to allow narrow-band radiofrequency detection methods to be used. Small changes in the particle's oscillation frequency will signal one-quantum transitions of the qubit, enabled by a quantum nondemolition (QND) coupling of the cyclotron and spin energies to ω_z.

We consider planar Penning traps formed from a spatially uniform magnetic field, $\mathbf{B} = B\hat{\mathbf{z}}$, and an electrostatic potential arising from applying biases V_i to N concentric metal electrodes with outer radii ρ_i in the $z = 0$ plane, with a grounded plane lying at $\rho > \rho_N$ and extending to infinity, depicted in Fig. 2. For appropriate choices of ρ_i and V_i, the potential on the $\rho = 0$ axis has a minimum at some point $z = z_0$. An expansion of the axial potential,

$$V(\tilde{z}) = \frac{1}{2} V_0 \sum_{k=0}^{\infty} C_k \, (\tilde{z} - \tilde{z}_0)^k, \tag{1}$$

yields the coefficients C_k that parameterize the potential. The overall scale V_0 is related to the axial oscillation frequency, $\omega_z^2 = qV_0/(m\rho_1^2)$, and distances $\tilde{z} = z/\rho_1$ are scaled by the radius of the innermost electrode. Standard electrostatics methods [27, 28] give analytical solutions for the potential in terms of the electrode radii and the potentials applied to them.

Any Penning trap realized in the laboratory has an electrostatic potential that differs from the pure quadrupole potential of an ideal Penning trap (i.e., $C_{k\geq 3} \neq 0$). Unlike cylindrical and hyperbolic traps, planar Penning traps are asymmetric about the trap center, so C_k are nonvanishing for odd k as well as even. These nonharmonic terms make the oscillation frequency depend on the amplitude, which undergoes thermal fluctuations due to coupling to an external circuit used for detection.

Minimizing the amplitude dependence of the axial frequency is the crucial design goal for planar Penning traps.

The Linstedt–Poincaré method [29] from classical perturbation theory can be used to derive a relationship for the axial frequency $\omega = \omega_z(\widetilde{A})$ as a function of oscillation amplitude $\widetilde{A}$ for the harmonic Fourier component, given by

$$\omega_z(\widetilde{A}) = \omega_z \left[1 + \sum_{k=2}^{\infty} a_k \widetilde{A}^k \right]. \tag{2}$$

The amplitude coefficients a_k depend on the potential coefficients C_k; the lowest-order terms are $a_2 = -15(C_3)^2/16 + 3C_4/4$ and $a_3 = C_3 a_2$. As a measure of the thermal width we will consider only the lowest-order contribution

$$\frac{\Delta\omega_z}{\omega_z} \approx |a_2| \frac{k_B T_z}{\frac{1}{2} m {\omega_z}^2 {\rho_1}^2}. \tag{3}$$

For small amplitudes $\widetilde{A} \ll 1$, minimizing the axial frequency width is achieved by designing a trap for which the lowest-order a_k vanish. This can be achieved either by making a trap for which the lowest-order C_k are zero, or by making a trap for which the lowest-order C_k are nonzero but their effects cancel each other out. For example, $a_2 = 0$ may be achieved either with $C_3 = C_4 = 0$ or with $C_4 = 5(C_3)^2/4$.

3 Optimized planar Penning traps

The simplest planar Penning traps have just two ring electrodes. However, no choice of ρ_i and V_i will achieve even $a_2 = 0$, so these traps are unsuitable for the sensitive detection methods required to realize a one-electron qubit.

For traps with $N = 3$, however, it is indeed possible to achieve $a_2 = a_3 = 0$ for certain choices of trap geometry, as shown in Fig. 3. For geometries to the right of the solid curve in Fig. 3, it is possible to optimize the trap using either the constraints $C_3 = C_4 = a_2 = a_3 = 0$ or $a_2 = a_3 = a_4 = a_5 = 0$ (the latter allowing $C_k \neq 0$), which in general each have two solutions with different values of z_0. However, there are particular geometries, indicated by the dashed curve in Fig. 3, for which these two possibilities can be satisfied simultaneously, leading to the remarkable cancellation $C_3 = C_4 = C_6 = a_2 = a_3 = a_4 = a_5 = 0$. These traps are expected to greatly suppress both the amplitude dependence of the axial frequency and the amplitude of higher harmonics of the fundamental mode of oscillation. A sample trap with $\rho_i = \{1.0909, 6, 8.2283\}$ mm (and hence $\tilde{\rho} = \{1, 5.5, 7.5426\}$) is shown in Fig. 2.

The expected performance of such a trap can be compared with the cylindrical traps used to observe single-quantum transitions by examining the calculated frequency-versus-amplitude curves, as shown in Fig. 4. The axial frequency shift is shown as a function of oscillation amplitude for the three optimized configurations of the sample trap. For one electron in the cylindrical trap an oscillation amplitude of 0.1 mm was large and easily detectable.

Planar Penning traps used in earlier experiments, indicated by crosses in Fig. 3, could not attain $a_2 = 0$ for any choice of trap potentials V_i. It is thus not so surprising that attempts to observe one electron in a planar Penning trap have not succeeded. The expected performance of these traps is considered in the appendices of Ref. [1].

Fig. 3 The *shaded regions* for which the indicated a_k can be made to vanish for a three-gap planar Penning trap, along with the region and the *curve* for which the indicated C_k can alternatively be made to vanish. To be avoided is the *shaded area* near the diagonal boundary $\tilde{\rho}_2 = \tilde{\rho}_3$ where there is a rather strong and sensitive cancellation between the effect of the potentials V_2 and V_3. No optimized traps are possible in the *unshaded region*, with the the earlier traps (*crosses*) at Mainz [12] and Ulm [14] as examples

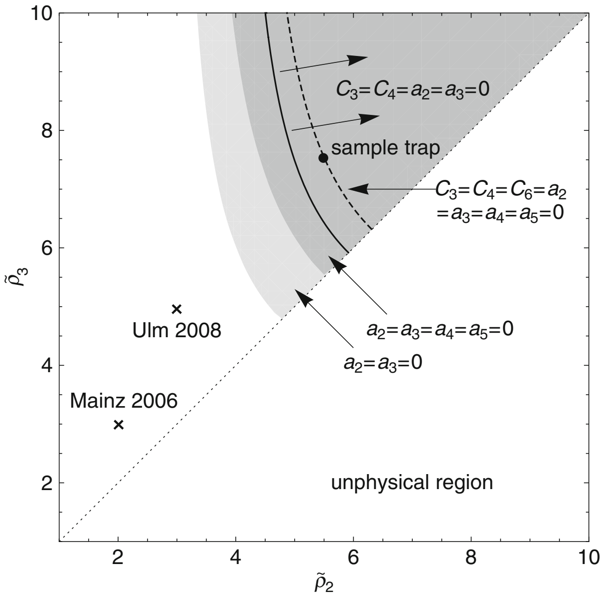
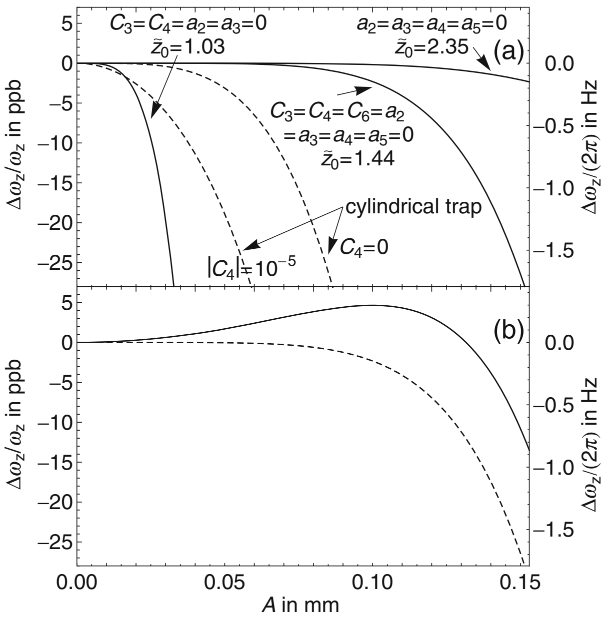


Fig. 4 **a** Amplitude dependence of $\omega_z(A) \approx 2\pi$ 64 MHz for optimized configurations of the sample trap is comparable or smaller than for the cylindrical trap. **b** Slight adjustments in the applied potentials minimize the dependence of ω_z upon fluctuations about a large oscillation amplitude (*solid*) [30] rather than for small oscillation amplitudes (*dashed*)



4 Laboratory Penning traps

Planar Penning traps put into service in the laboratory will not have the ideal properties described in the previous sections of this work. Small gaps of some width w between electrodes are unavoidable but variations in the potential should be small

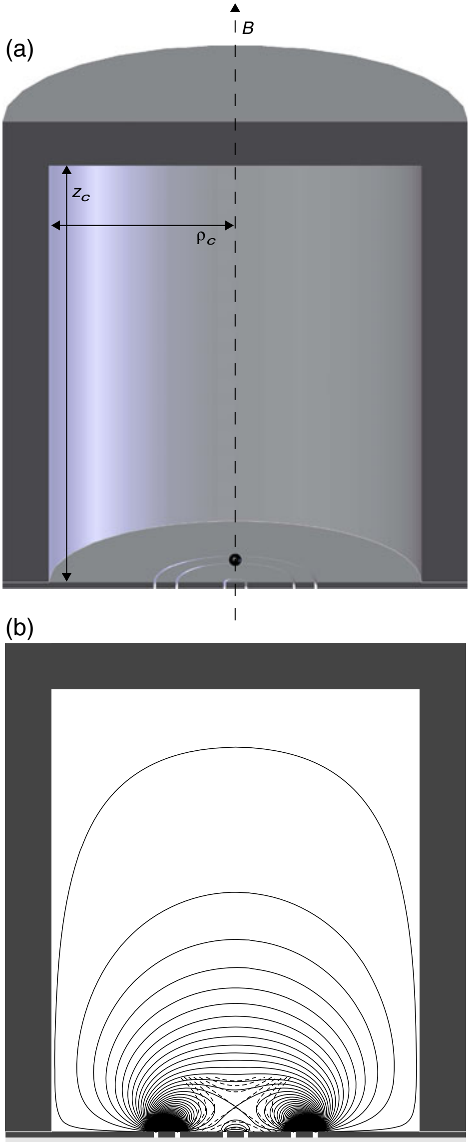


Fig. 5 **a** Planar trap enclosed within a conducting, capped cylinder. Particles can be loaded through a tiny axial hole in the cover (not visible). **b** Side view of trap electrodes and equipotentials spaced by V_0, with the infinitesimal gaps between the electrodes widened to make them visible. The equipotentials extend into the gaps between electrodes. The dashed equipotentials of an ideal quadrupole are superimposed near the trap center

as long as $w \ll z_0$. By approximating the potential in the gaps as varying linearly between the potentials of the neighboring electrodes [16], we find that the same optimization conditions can be satisfied but with applied potentials that are slightly shifted.

There are two additional important considerations associated with gaps: First, charges that accumulate on the insulators in the gaps can substantially modify the trapping potential. Careful loading and operation procedures and high-aspect-ratio gaps can minimize these effects. Second, currents can flow between polished electrodes separated by small gaps. These field emission currents [31–34] grow exponentially with the difference in potential across the gap. Trap designs that limit the size of the gap potentials are one solution. Other solutions are to increase the gap width and to make the metal surfaces within the gap as smooth as possible. As planar traps and planar trap arrays get smaller it will be necessary to investigate these solutions further.

For laboratory traps it is difficult to keep all parts of the apparatus many trap diameters away from the trapping volume. The effects of realistic finite boundary conditions are thus extremely important.

One choice of finite boundary conditions comes from locating a planar trap within a grounded conducting cylinder closed with a flat plate (Fig. 5). By calculating the potential from the solution to LaPlace's equation including the conducting walls, it is possible to achieve the same optimized trap properties by applying slightly different biases than infinite-boundary case.

A fabricated laboratory trap will not have exactly the intended dimensions and hence the intended properties because of unavoidable fabrication imprecision. The solution must be to slightly adjust the potentials on the electrodes to recover properties closer to the ideal, if this is possible. For imprecision in the trap radii the effective radii for the electrodes will be unknown, so we need a way to tune the trap in situ to make $a_2 = 0$. The other important coefficients will remain small enough because of the optimized design. Changing the potential on each electrode will change both a_2 and ω_z. The result is that it is necessary to adjust two or three of the potentials applied to the electrodes in such a way that a_2 varies by a reasonable amount while keeping V_0 and ω_z fixed.

5 Covered and mirror-image Penning traps

A covered planar Penning trap (Fig. 6) is a planar trap that is electrically shielded by a nearby conducting plane. The covered planar trap has some very attractive features.

1. The electrodes are in a single plane that can be fabricated as part of a single chip.
2. The conducting plane provides an easily controlled boundary condition above the electrode plane that needs no special fabrication, nor any alignment beyond making the planes parallel.
3. A trap that is radially infinite is well approximated if the radial extent of the two planes beyond the electrodes is large compared to their spacing.
4. A covered planar trap is naturally scalable to an array of traps.
5. The axial motion of electrons in more than one trap could be simultaneously detected with a common detection circuit attached to the cover.
6. The axial motions of electrons in more than one trap could be coupled and uncoupled as they induce currents across a common detection resistor by tuning the axial motions of particular electrons into and out of resonance with each other.

Three possible additional advantages emerge when the properties of the trapping potential in a covered planar trap are considered.

1. A two-gap covered planar trap can be optimized in much the same way as a three-gap infinite planar trap. Figure 8a shows the geometries for which a two-gap planar trap can be optimized.

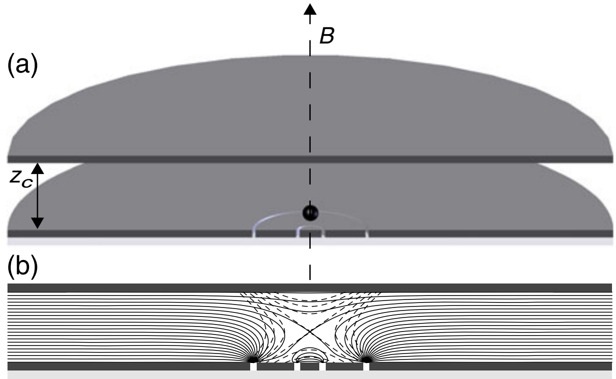


Fig. 6 **a** A covered planar Penning trap, which could be loaded through a tiny axial hole in the cover (not visible). **b** Side view of trap electrodes and equipotentials spaced by V_0, with the infinitesimal gaps between the electrodes widened to make them visible. Some equipotentials extend into the gaps between electrodes and some terminate at infinity. The dashed equipotentials of an ideal quadrupole are superimposed near the trap center

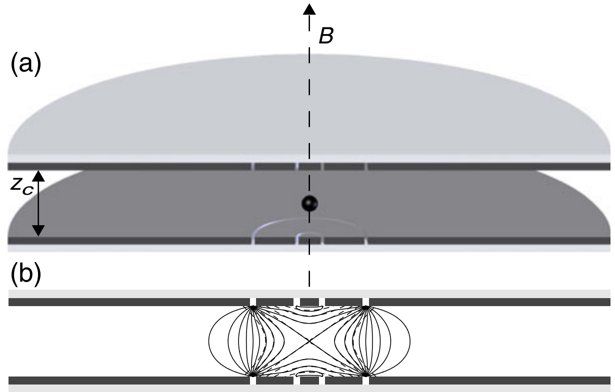


Fig. 7 **a** A mirror-image Penning trap is formed with two planar trap electrodes facing each other. Particles can be loaded through a tiny axial hole in one of the electrodes (not visible). **b** Side view of trap electrodes and equipotentials spaced by V_0, with the infinitesimal gaps between the electrodes widened to make them visible. The equipotentials extend into the gaps between electrodes. The dashed equipotentials of an ideal quadrupole are superimposed near the trap center

2. Smaller gap potentials can sometimes be used to achieve optimized configurations, permitting smaller gap widths and better screening of the exposed insulator between electrodes.
3. In some cases a smaller a_6 can be realized for trap configurations with $a_2 = a_3 = a_4 = a_5 = 0$.

A mirror-image planar trap (Fig. 7a) is a set of two planar electrodes that are biased identically and face each other. The properties of a mirror-image trap are similar to those of the cylindrical Penning trap (Fig. 1c) used to suspend one electron and to observe its one-quantum cyclotron transitions and spin flips. A charged particle suspended midway between the two electrode planes sees a potential that

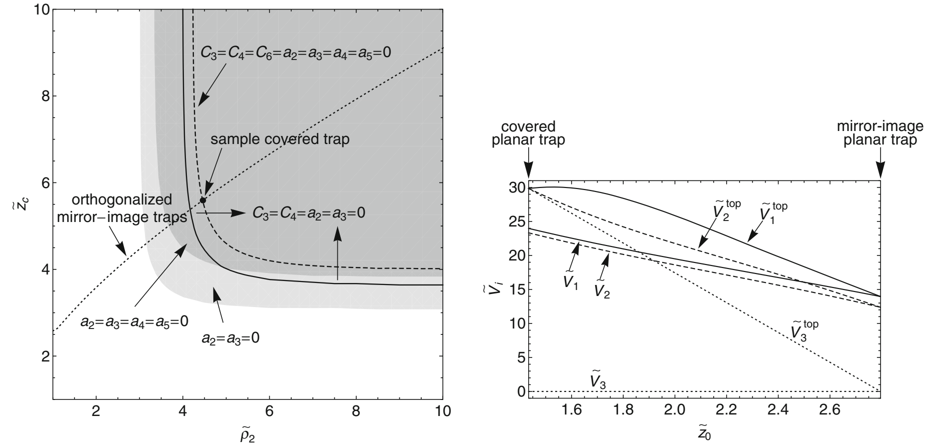


Fig. 8 **a** Parameter space regions for which the indicated a_k can be made to vanish for a two-gap planar trap with a cover, along with the region and the curve for which the indicated C_k can alternatively be made to vanish. No optimized traps are possible in the unshaded region. The *dotted line* indicates orthogonalized mirror-image traps formed from two sets of two-gap planar trap electrodes. Only one relative geometry can realize either a harmonically optimized covered planar trap and or an orthogonalized mirror-image planar trap; the properties of such a trap are further considered in Ref. [1]. **b** One set of applied potentials that relocates an electron centered between the electrode planes of a mirror-image planar trap (*far right*) to a covered planar trap (*far left*) while keeping the axial frequency constant and keeping $a_2 = a_3 = C_3 = C_4 = 0$

is symmetric under reflections across this midplane, in which case all odd-order potential coefficients (C_3, C_5, etc.) vanish, as for the cylindrical trap. Also, as for a cylindrical trap, we can choose the potentials applied to the trap electrodes to make a trap with a very small C_4, whereupon a_2 and a_3 are very small.

A useful property of mirror-image traps and cylindrical traps is that both of these can be “orthogonalized” [18, 19] in a way that a planar trap cannot. A single potential (applied to two electrodes with mirror-image symmetry) is tuned to minimize the amplitude-dependence of the axial frequency. The trap is orthogonalized in that this tuning does not change the axial frequency, which in general would take it out of resonance with the detection circuit.

At least for initial studies it may be useful first to load an electron into the center of an orthogonalized mirror-image trap. The presence of a single electron can be established with established methods used with cylindrical Penning traps. The potentials applied to the electrodes can then be changed adiabatically to turn the mirror-image trap into a covered planar trap. The potentials in Fig. 7b indicate one choice of intermediate potentials for which the trap remains optimized during every point in the transfer, with $a_2 = a_3 = C_3 = C_4 = 0$. It may thus be possible to detect the electron’s axial oscillation at every step of the transfer.

6 Conclusion

A cylindrical Penning trap has been used to observe one-quantum spin flip and cyclotron transitions of a single trapped electron. Attempts to make similar observations in a planar Penning trap did not succeed, generating some pessimism about whether this is possible. We present new designs that optimize the properties of a planar Penning trap to drastically minimize the amplitude dependence of the monitored axial frequency by orders of magnitude. We introduce a covered planar trap that is well isolated from its environment, readily scalable to an array of one-electron traps, with one detector promising to suffice for the efficient simultaneous detection of multiple particles. We also introduce mirror-image planar traps that are an attractive option because of their reflection symmetry. A mirror-image trap can be electrically transformed into a covered planar trap while a particle is stored within. The optimized planar trap designs that are proposed offer new routes toward observing a single electron in a planar trap, realizing a one-electron qubit, and using a scalable array of such qubits for quantum information studies.

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