Special Relativity and the Single Antiproton: Fortyfold Improved Comparison of $\bar{p}$ and p Charge-to-Mass Ratios

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The measured ratio of charge-to-mass ratios for the antiproton and proton is $1.000\,000\,001\,5 \pm 0.000\,000\,001\,1$. This 1 part in 10^9 comparison (1 ppb) is possible because a single $\bar{p}$ or p is now directly observed while trapped in an open access Penning trap. The comparison is the most accurate mass spectrometry of particles with opposite charge, and is the most sensitive test of CPT invariance for a baryon system. It is 40 times more accurate than our earlier comparison with many trapped antiprotons and protons, and is more than 45 000 times more accurate than earlier comparisons made with other techniques.

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At its discovery [1], the antiproton ($\bar{p}$) was identified by comparing its charge-to-mass ratio (q/M) to that of the proton (p). The accuracy of the mass comparison (Fig. 1) increased when transition energies were measured for antiprotons orbiting as "heavy electrons" in exotic atoms [2–5]. The charge-to-mass ratios for $\bar{p}$ and p were compared more than 1000 times more accurately when our TRAP collaboration developed the slowing, trapping, cooling, and stacking techniques [6,7] to reduce by 10^{10} the energy of 5.9 MeV antiprotons from the unique LEAR facility of CERN, yielding more than 10^5 trapped $\bar{p}$ at 4.2 K. The cyclotron frequencies $\nu_c = qB/2\pi M$ of approximately 100 trapped antiprotons and protons were compared to 4×10^{-8} in the same magnetic field B [8].

Comparing a measured ν_c for a single trapped $\bar{p}$ and p now makes it possible to compare their charge-to-mass ratios 40 times more accurately, an improvement by a factor of 45 000 over the exotic atoms measurements. Special relativity is crucial in that ν_c depends upon the "relativistic mass" $M = \gamma M_0$, where $\gamma = 1/\sqrt{1 - v^2/c^2}$ is the familiar function of the particle's speed v normalized to that of light c [9,10]. Detected cyclotron excitations of 1 to 200 eV are such low kinetic energies as to be generally regarded as exceedingly nonrelativistic, with $\gamma < 1.000\,000\,2$. However, the cyclotron frequency of one trapped $\bar{p}$ or p is measured with a resolution so high ($<2 \times 10^{-10}$) that the relativistic frequency shift is inescapably large, providing an especially clean demonstration of special relativity along with the greatly improved comparison of $\bar{p}$ and p.

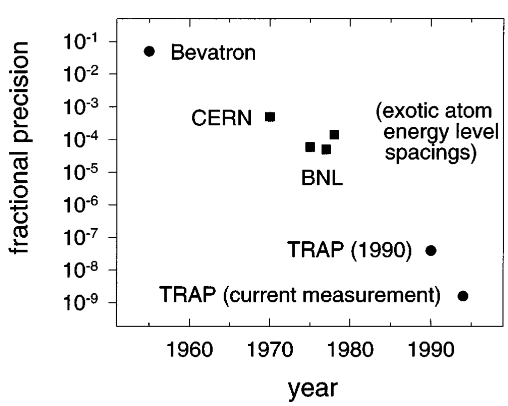

FIG. 1. Comparisons of charge-to-mass ratios (circles) and inertial masses (squares) for $\bar{p}$ and p.

By 4 orders of magnitude, the new measurement is the most precise test of CPT invariance made with baryons, with C, P, and T representing charge conjugation, parity, and time reversal transformations. The invariance of physical laws under CPT transformations is widely assumed to be true, despite the possibility to violate P, CP, and T separately, because it is not possible to construct a Lorentz invariant, local field theory which is not invariant under CPT [11]. Such invariance implies that the inertial masses and charge magnitudes of a particle and antiparticle are identical, along with their mean lives and magnetic moment magnitudes. Despite the fundamental importance of CPT invariance, precise experimental tests are very scarce [12]. Only one lepton magnetic moment comparison (of e^+ and e^- [13]) and one meson mass comparison (of K_0 and $\bar{K}_0$ [14]) are of comparable or higher fractional precision than the baryon comparison reported here.

Antiprotons, obtained at 5.9 MeV from the low energy antiproton storage ring (LEAR), have their energy reduced to 0.3 milli-eV within our apparatus. They slow below 3 keV in a degrader and are caught in a Penning trap [6], then cool via collisions with cold electrons in the trap [7] to thermal equilibrium at 4.2 K. Typically, 10^4 antiprotons reside with approximately 10^7 electrons in the Penning trap. To selectively eject the electrons, which would otherwise disrupt the precision comparison of $\bar{p}$ and p, the voltage on one end of the trap is pulsed to open the trap for 200 ns. The antiprotons remain while the less massive electrons escape.

The open access Penning trap [15] provides a good environment for precision mass spectrometry along with the access needed to initially admit antiprotons before cooling. It consists of a 5.85 T magnetic field (from a persistent superconducting solenoid) and a superimposed electrostatic quadrupole potential. Trapped particles have three oscillatory motions [10]. The axial motion, at frequency $\nu_z = 954$ kHz, is along the direction of the magnetic field. The trap-modified cyclotron motion, at a higher frequency $\nu_c' = 89.3$ MHz, is a circular motion in a perpendicular plane, as is the magnetron motion at a much lower frequency $\nu_m = 5.1$ kHz. Unlike traditional traps used for precision mass measurements (with electrodes shaped along the hyperbolic equipotentials of the desired quadrupole potential), this trap is made of stacked cylinders, each with the same inner diameter [Fig. 2(a)]. A careful choice of electrode lengths [15] and careful tuning of the applied voltages produces the high quality electrostatic quadrupole needed to produce harmonic motions, with frequencies independent of excitation energy. The observed signal to noise in this and related cylindrical configurations is as good as that observed in the hyperbolic traps. A key feature is an orthogonality which keeps the well depth from changing during the tuning. The trap is within a sealed vacuum enclosure kept at 4.2 K by thermal contact to liquid helium, to produce a vacuum that our earlier $\bar{p}$ measurements indicate is better than 5×10^{-17} Torr [8]. (This vacuum allowed storing two antiprotons for 60 days.)

The cyclotron and axial motions of a trapped $\bar{p}$ or p are observed when they induce detectable oscillatory voltages across attached LCR circuits [Fig. 2(a)]. Energy dissipation in the two circuits damps these motions into thermal equilibrium with the tuned circuits near 4.2 K. To maximize the signal and damping, the quality factor (Q) for each circuit is made as large as possible. A circuit resonant at 89.3 MHz with $Q = 800$ is connected to one of four ring sections to detect the cyclotron oscillation. The axial motion is detected similarly except that a nearly resonant driving potential is applied to the endcap opposite the axial detection circuit. A superconducting inductor and shield (made of NbTi to permit operation in the 5.85 T field) allow a high $Q = 3000$ at 954 kHz.

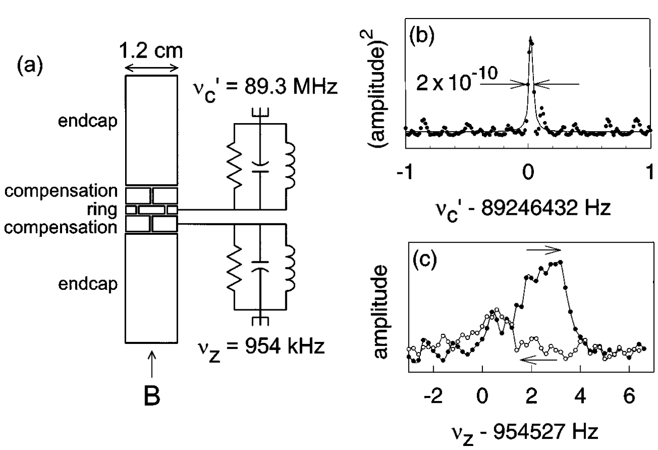

FIG. 2. Open access Penning trap electrodes and detection circuits in (a), with the cyclotron (b), and axial (c) signals from one trapped $\bar{p}$.

To reduce the number of trapped antiprotons to one, the cyclotron motion of the antiprotons is excited by a strong, nearly resonant drive pulsed onto a segment of the ring or a compensation electrode. The broad cyclotron response signal is monitored as the trapping well depth is reduced from 18 eV to below 0.3 eV to spill antiprotons. This signal breaks into separate resonance peaks when less than 15 antiprotons remain; each peak is due to an excited antiproton with a distinct cyclotron energy and thus a distinct cyclotron frequency because of special relativity. The trapping potential is lowered until only one antiproton is still observed, then restored to 18 V. The trapped particle is radially centered by a strong sideband cooling drive at frequency $\nu_z + \nu_m$ [16] applied to one-half of a split compensation electrode.

One proton is loaded with the trapping potential on the ring switched to -18 V. A keV electron beam from a field emission point (inside the trap's vacuum enclosure) is sent through the trap to strike a surface. Some atoms liberated from this surface collide with the electron beam within the trap volume, are ionized, and become trapped. Strong axial noise at frequencies below 850 kHz is applied to one endcap to drive out positive ions which would otherwise load into the trap. (Even one remaining ion prevents an accurate measurement.) Notch and low pass filters reduce this noise by 120 dB at $\nu_z = 954$ kHz to prevent driving out a proton. We alternately drive and detect at the cyclotron frequency ν_c', switching off the electron beam when a cyclotron signal indicates that one proton is trapped.

The large, undriven cyclotron signal of one trapped $\bar{p}$ [Fig. 2(b)] has a frequency resolution narrower than 2×10^{-10}, limited by the Fourier transform width of the detector. (The measured decay time discussed below corresponds to a much narrower width.) Figure 2(c) shows driven axial signals for one $\bar{p}$ which differ when the driving force is swept upward and downward in frequency because the trap is not quite tuned to produce a perfect electrostatic quadrupole.

An initially excited cyclotron motion damps exponentially by slowly dissipating energy in the detection circuit. Special relativity shifts ν_c' upward in proportion to the remaining excitation energy E_c, as illustrated by three cyclotron resonances at different times [Fig. 3(a)]. The time

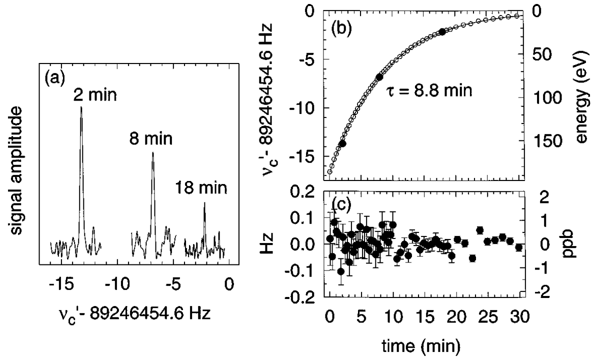

FIG. 3. Special relativity shifts the cyclotron frequency of a single trapped $\bar{p}$ as its cyclotron energy is slowly and exponentially dissipated in the detector. Cyclotron signals for three subsequent times in (a) have frequencies highlighted in the measured frequency vs time points in (b). A fit to the expected exponential has small residuals (c) and gives ν_c' for the limit of no cyclotron excitation.

dependent ν_c' [Fig. 3(b)] is fit to the expected exponential to extract the ν_c' endpoint, the limiting value for vanishing E_c, and the residuals [Fig. 3(c)] are small. Quadratic gradients in the magnetic field (i.e., a "magnetic bottle") and electrostatic anharmonicity similarly couple ν_c' and E_c, but much less strongly.

The compared cyclotron frequency $\nu_c = qB/(2\pi M)$ differs slightly from ν_c' (the cyclotron frequency in the trap), but is related to the three measured frequencies ν_c', ν_z, and ν_m by the invariance theorem [17]

$$(\nu_c)^2 = (\nu_c')^2 + (\nu_z)^2 + (\nu_m)^2, \qquad (1)$$

which is independent of the leading perturbations of an imperfect Penning trap (e.g., tilts of the magnetic field and quadratic changes in the trapping potential). Both ν_c' (from a decay endpoint) and the axial frequency ν_z are always measured. Attaining a 1 parts per 10^9 (ppb) accuracy in ν_c requires a careful measurement of ν_z to better than 8 Hz, but ν_m needs only to be measured to 10% and is thus measured less often.

The 5.85 T magnetic field fluctuates in time because the ambient magnetic field (in which the solenoid is located) is fluctuating. While high frequency fluctuations are shielded by eddy currents induced in various cylindrical conductors surrounding the trap, low frequency fluctuations are potentially very serious. Magnets from the nearby CERN proton synchrotron (PS) are the largest problem, making 4 μT (40 mG) fluctuations at our location as often as every 2.4 sec. The solution is to cancel such fluctuations at the location of the trapped particles by the addition of a superconducting solenoid inductively coupled to the high field solenoid [18]. Currents induced in the coupled superconducting solenoids cancel the effect of spatially uniform fluctuations by a factor of 156 [19], without compromising the homogeneity of the magnetic field. Gradients in the fluctuating fields from nearby sources reduce the shielding of the PS fluctuations to a factor of 110 and the LEAR magnets only several meters away are shielded by a factor of 50. Fluxgate magnetometers monitor the ambient field during a measurement to alert us to external magnetic fluctuations too large to be canceled by the self-shielding solenoid system.

Magnetic field stability remains a problem even when ambient fluctuations are eliminated. Using a $\bar{p}$ or p as a magnetometer over many days shows that the magnetic field varies slowly depending upon the pressure and boil-off rate for the helium reservoirs which cool the superconducting solenoid and keep the trap at 4.2 K. The pressures are thus monitored and regulated over these reservoirs and the solenoid's nitrogen reservoir, and gas flows from the dewars are monitored. Nonetheless, a daily drift in the magnetic field [visible in Fig. 4(a)] correlates with unfortunately large changes in the temperature of the accelerator hall. Such field drifts are slow enough ($<$2 ppb/hr) to fit to a quadratic or cubic function of time, provided that the temperature regulation for the pressure reference volume does not go out of range. (Two complete p-$\bar{p}$-p comparisons were lost in this way.)

One of the four p-$\bar{p}$-p comparisons which comprise this measurement is shown in Fig. 4(a). The four points to the left are the measured cyclotron frequencies ν_c (each from a fitted ν_c' endpoint and a measured ν_z) for four cyclotron excitations of the same trapped p. A single $\bar{p}$ was then loaded in place of the p, and ν_c' was measured for three cyclotron decays (triangles). A p was again loaded and ν_c measured for several more cyclotron decays. All ν_c values were then fitted to cubic functions of time as mentioned, with a possible difference $\Delta\nu_c = \nu_c(\bar{p}) - \nu_c(p)$ included as a fitting parameter. Figure 4(b) shows this difference (in ppb) for the four p-$\bar{p}$-p comparisons. The weighted average and the standard deviation of the points, divided by $\sqrt{4}$, are given by $\Delta\nu_c = 1.5 \pm 0.3$ ppb. (The average decreases by 0.3 ppb if quadratic fits are used instead.)

The largest measurement uncertainty arises because the $\bar{p}$ and p have opposite sign of charge, and thus require externally applied trapping potentials of opposite sign. Reversing the applied potential does not completely reverse the potential experienced by the particle (e.g., due to the patch effect and charges on the inner surfaces of the trap electrodes). During a mass measurement the $\bar{p}$ and p thus reside at slightly different locations, as if an

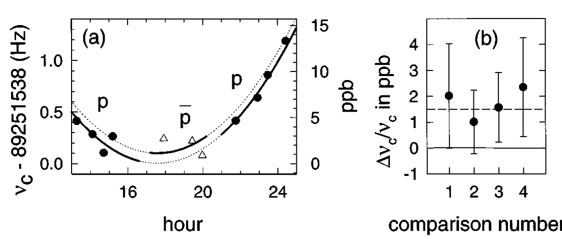

FIG. 4. (a) Measured ν_c for one p, one $\bar{p}$, and then a second p. (b) Fractional differences in ν_c for $\bar{p}$ and p, from four p-$\bar{p}$-p comparisons like that in (a), before correcting for differing locations of $\bar{p}$ and p.

unchanging offset potential was applied to trap electrodes to either side of the particle. If the nearly homogeneous magnetic field differs slightly between the two locations, ν_c for $\bar{p}$ and p will differ even if the charge-to-mass ratios do not.

Prior to the p-$\bar{p}$-p comparisons, the current in nine superconducting shim coils was adjusted in several iterations, to minimize the magnetic gradients. The first iteration was done with an NMR probe in place of the particle trap. However, subsequent iterations required a single $\bar{p}$ or p as the probe in order to avoid the extremely small but important magnetism of the NMR probe and the trap. We move $\bar{p}$ and p away from their measurement locations in three orthogonal directions (by applying offset voltages across the endcaps, and across opposite ring quadrants). The measured ν_c as a function of position for each particle reveals the magnetic field gradients. Unfortunately, a "large" nearly linear gradient of 20 ppb/mm remains in one radial direction (ten times larger than the gradients in the orthogonal directions). With more iterations, the large gradient could be reduced, but this long and tedious process takes weeks. The shims are not completely orthogonal and the trap must be retuned at every particle location to make the axial oscillation harmonic enough to measure ν_z. The gradients and relative particle locations did not change noticeably while the trap remained at 4.2 K as long as no cooling electrons hit the trap electrodes.

We estimate that the radial separation between the equilibrium measurement locations for the $\bar{p}$ and p can be no larger than 50 μm. This corresponds to an uncertainty of 1 ppb and to an effective 0.2 V offset potential applied across opposing segments of the ring electrode. (We have observed an offset this large when we mistakenly permitted charged particles to hit the electrodes of our trap.)

The ratio of the antiproton and proton charge-to-mass ratios (expressed as the mass ratio which is traditional for mass spectroscopy) is thus given by equation

$$M(\bar{p})/M(p) = 0.999\,999\,999\,5(11), \qquad (2)$$

with the uncertainty in the last digits in parentheses. As discussed earlier, this ratio represents the most accurate mass spectroscopy of particles of opposite sign and is the most accurate test and confirmation of *CPT* invariance with a baryon system.

For the future, we will compare ν_c for an H^- ion and a $\bar{p}$ stored together in the same trap. The dual advantages of comparing species with the same charge sign, and more rapid switching between species, should allow a precision similar to the 0.1 ppb which has been attained with positive ions loaded for comparison every few minutes [20]. It should thus be possible to deduce an even more accurate comparison of $\bar{p}$ and p despite the special challenges which pertain for mass spectroscopy on an exotic species at an accelerator facility.

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