

**Parametric Dark Detection
Of A Single Electron
In A Penning Trap**

A thesis presented

by

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Abstract

The gyromagnetic ratio measurement of the electron is the most precise comparison between a theory (QED) and an experiment. The leading systematic error is due to the uncertainty of the coupling between the electron and the surrounding trap cavity. To overcome this, the microwave cavity mode structure of a cylindrical geometry trap has been characterized by using parametrically-driven electron oscillators. The coupling between the electron and the radiation field environment of the cavity can now be controlled and measured directly.

The parametric oscillation of a single trapped electron is studied and used to measure enhanced spontaneous emission, an emission rate which is much faster than could be measured before. Hysteresis in this motion provides a 1 bit memory to store information about excitations made with the electron "in the dark". Using this technique, the cyclotron frequency is measured to the precision (1 ppb) needed for a new g factor measurement. A one electron, self-excited oscillator using parametric feedback is examined. The exponential risetime of a parametric excitation is measured, and may provide a sensitive means by which to measure the temperature of the particle.

The cyclotron excitation of one electron is modelled as a driven, damped, anharmonic oscillator at non-zero temperature. The master equation and the associated Q -function are presented. Cyclotron excitations as large as 16.2 eV ($n=26,500$) were observed. Rates for possible cavity mode sideband cooling of the axial temperature are estimated.

The cyclotron excitation of two electrons is also observed, and the basic features explained using a simple model. Multi-particle plasma interactions can thus be studied in one particle increments.

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3. "Extremely Cold Antiprotons For Mass Measurements and Antihydrogen", G. Gabrielse, W. Jhe, D. Phillips, C. Tseng, L. Haarsma, K. Abdullah, J. Gröbner, H. Kalinowsky, in *Atomic Physics 13, Thirteenth International Conference on Atomic Physics*, (American Institute of Physics, New York), 85 (1993).
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7. "One Electron Parametric Oscillator", C.H. Tseng and G. Gabrielse, (festschrift for Prof. Dr. H. Walther), *Appl. Phys. B*, **60**, 95 (1995).

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