Chapter 1

Introduction

Once upon a time, there was an old cottar who had several children, but was too poor to feed and clothe them [3]. One day, a big, big, white bear came and offered to exchange riches of gold and silver for his fair, youngest daughter. The bear took her to a marvelous palace far, far away. Transformed, he would visit her at night only when she had blown out the last light, but would leave before daybreak. One night she hid a candle in her bosom, and discovered that it was indeed a handsome young prince who lay beside her. Three drops of hot tallow fell and woke him. Compelled to return to the castle east of the sun and west of the moon, he scolded her for not having patience. For within a year, he was to be freed from the spell cast by his evil stepmother. With the aide of the Four Winds, the young maiden sought out her prince in the castle that was farther than far. She saved the prince from the evil stepmother, an old troll woman, who threw such a rage that she burst. Presumably, the maiden and the prince lived happily ever after.

From this Norwegian fairy tale we learn that measurements not performed in the dark may cause serious disturbances. Similarly, high-resolution spectroscopy measurements require a large, easily detectable signal while at the same time requiring an unperturbed system. For the series of determinations of the electron and positron anomalous magnetic moments [85, 86, 82, 21, 25], as well as for the broader class of precision spectroscopy measurements on trapped ions, it is becoming increasingly more desirable to reduce the motions of the particle even below the level of disturbances due to the final detection process. To this end, a detection scheme is demonstrated that utilizes the bistability and hysteresis of a parametric oscillation of a single trapped electron to provide a 1 bit memory to store information about excitations made with the electron "in the dark". That is, the detection electronics, which would otherwise heat up the motions of the particle, are turned off during the measurement. During the crucial portion of the cyclotron excitation and measurement process, there are no additional drives that are able to make any appreciable amplitude excitation. The 1 bit memory is subsequently read out by a very large signal that has been parametrically amplified. This process of parametric dark detection is used to measure the unperturbed cyclotron frequency of the electron to a 1 ppb precision. This is the sensitivity needed to distinguish the spin state of the electron, the crucial determination for the magnetic moment measurements.

The magnetic moment, $\vec{\mu}$, of the electron is related to its spin $\vec{S}$ by the gyromagnetic ratio, or g factor.

$$\vec{\mu} = g\left(\frac{-e}{2mc}\right)\vec{S}, \tag{1.1}$$

where -e is the electron charge. This factor expresses the interaction of the electron with a magnetic field, and is characteristic of its internal structure. The simplest Dirac equation predicts a value for g of 2 for the electron or positron. (For the proton, the value is 5.59, indicating a complicated hadronic internal structure.) Slight corrections (0.1 percent) due to radiative corrections are described by quantum electrodynamics, the celebrated theory of Tomonanga, Schwinger, Feynman, and Dyson. This theory, describing the interaction between leptons and the photon, is typically formulated using the fundamental and dimensionless fine structure constant, α. These radiative corrections are quantified by the anomaly,

$$a \equiv \frac{g-2}{2}. \tag{1.2}$$

The anomaly to first order in α, derived by Schwinger in 1948, is $a = \alpha/2\pi = 0.00116$, and has since been calculated to fourth order in α (eighth order in e) by Kinoshita [57, 58, 59, 60, 61]. Very recently, the theoretical calculation has exceeded the precision of the experimental result, but only by a small amount. The gyromagnetic ratio is in fact the most stringent comparison between a theory and an experiment. The values are [85, 58],

$$\begin{aligned} a_{expt}(e^-) &= 1\,159\,652\,188.4(4.3) \times 10^{-12} \\ a_{expt}(e^+) &= 1\,159\,652\,187.9(4.3) \times 10^{-12} \\ a_{theory} &= 1\,159\,652\,137.3(3.5)(2.1)(27.1) \times 10^{-12}. \end{aligned} \tag{1.3}$$

The errors in the theoretical number are respectively from the error in the α^3 term; the error in the α^4 term; and the error in the value of α measured from the quantized Hall effect. The theoretical value is within 1.9 standard deviations of the experimental value. This has led to the suggestion that the g factor measurement be used to determine the fine structure constant. Also, the comparison between the electron and positron g factors [85] is the most exacting test of charged lepton-antilepton symmetry (exceeded only by the comparison of the neutral kaons, K_0 and $\bar{K}_0$ [93]). The most precise test of this symmetry in a baryon system is the proton-antiproton comparison [43]. The g factor can be written as the ratio of the spin precession and cyclotron frequencies,

$$g/2 = \omega_s/\omega_c. \tag{1.4}$$

Exploiting the fact that the two frequencies are nearly degenerate (0.1 percent), the precision of the determination is increased by measuring the anomaly frequency, $\omega_a = \omega_s - \omega_c$, instead [22],

$$a \equiv \frac{g-2}{2} = \frac{\omega_s - \omega_c}{\omega_c} = \frac{\omega_a}{\omega_c}. \tag{1.5}$$

Thus, the anomaly a and the g factor can be determined by measuring the anomaly frequency and the cyclotron frequency. The anomaly transition, which is a simultaneous spin flip and a cyclotron transition, can be detected as a shift in the cyclotron frequency (at the level of 1 ppb). Therefore, a precise method of determining the cyclotron frequency is essential for a measurement of the g factor.

The cyclotron motion, at frequency $\nu_c \approx 150$GHz in our 5.3 Tesla magnetic field, is difficult to observe directly. In the past, the cyclotron motion was coupled to the detected axial motion of the electron through a deliberate magnetic gradient known as a magnetic bottle [84]. This allows a small cyclotron excitation to be detected as a shift in the axial frequency. Typical excitations are very small (about 5 quantum levels in the cyclotron Landau ladder). This bottle unfortunately introduces significant lineshape broadening because the electron moves through an inhomogeneous magnetic field [9, 10]. We instead use the small coupling due to special relativity. This "relativistic bottle" does not introduce a magnetic field gradient, but still does couple the axial and cyclotron motions [39]. This is described more in Chapter 4. The effective coupling strength about 30 times smaller than that provided by the external magnetic bottle used in the the latest g factor determination. The cyclotron motion is excited to a much higher level, with a correspondingly larger signal [39, 71, 72], by making use of the high Q$\approx 10^{10}$ and weak anharmonicity. This serves as a sort of amplifier, that is, the large excitation is determined by the ground state properties. Combinations of both types of coupling have been tried [68, 69, 26, 39] in an effort to minimize the reaction of the detection noise back on the cyclotron motion. Effective temperatures of similar detectors have been estimated to be about 16 K [85], and even up to 130 K [26].

The leading systematic error limiting the last measurement of the electron g factor [85] is due to the uncertainty of the coupling between the electron and the radiation modes of a surrounding trap cavity [6, 7, 12, 13, 15]. To understand this, a cylindrical geometry trap was designed [42, 77, 14] that has an easily calculable cavity mode structure. This cavity environment was studied by using parametrically-pumped electron oscillators [78, 76], allowing the characterization of the radiation field environment of the electron. A new technique, described in Chapter 3, uses the parametric oscillation of a single trapped electron to measure the cavity modified damping rate due to inhibited or enhanced spontaneous emission for a wider range of emission rates than was possible before. The ability to measure a very fast decay constant is especially important when the cyclotron frequency is tuned to match the resonant frequency of a cavity mode. In this case, the coupling is typically so strong that the rapid damping rate could not have been measured by the previous method. The coupling of the electron to the cavity can now be controlled and measured directly with one electon *in situ* for a broad range of values of decay times. This should permit a more precise measurement of the electron g factor.

In Chapter 2, the apparatus is discussed. There are general requirements of homogeneity and stability that apply to each of the subsystems such as the magnetic field, the electric quadrupole potential, the cryostat, and the cyclotron microwave drive. The orthogonalized, hyperbolic Penning trap that was constructed [35] provides an exceptional signal-to-noise ratio, and allows "noise shorting" to be clearly observed for a single electron in a Penning trap. A curious demonstration is also discussed wherein trapped electrons were transported across the continental United States [80]. This is of interest for moving antimatter for precision measurements, and has been suggested for use in a possible method for medical imaging and treatment [48, 49, 54].

Chapter 3 presents the parametric oscillator. The characteristics are studied and compared with the simple classical model for a parametric oscillator. The features are exploited for several new techniques. For example, the lineshape is fitted to extract the anharmonicity coefficients that characterize the quadrupole trapping potential. The method of detecting fast enhanced spontaneous emission is discussed and illustrated. Parametric dark detection is explained, and used to measure the cyclotron frequency to a precision of 1 ppb. The exponential parametric excitation risetime may be a way to measure the particle's temperature. Finally, a parametric feedback loop is used to demonstrate a self-excited monoelectron oscillator.

Chapter 4 is a theoretical treatment of the relativistic anharmonic cyclotron oscillator. The density operator master equation for the driven, damped, anharmonic oscillator at non-zero temperature is presented and discussed. Some experimental observations are compared with theoretical expectations. An example of a high resolution cavity mode is shown. Rates for possible sideband cavity cooling of the motion of the particle are estimated. The cyclotron excitation of two electrons in a Penning trap is also observed, and the basic features explained using a simple model. The more complicated features may indicate order-chaos transitions [52, 8, 34, 5, 4]. This sensitivity demonstrates that it is possible to study multi-particle plasma interactions in one particle increments. This is similar to the studies of two ions above, or the proton-antiproton system, or the antiproton-H^- ion system [41, 43].

Chapter 5 discusses the prospects for future development of these techniques, with an emphasis on a more precise measurement of the electron g factor.