

**Cooperative Behavior
in
Cavity-cooled,
Parametrically-pumped
Electron Oscillators**

A thesis presented

by

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Abstract

Stochastic motions of parametrically-pumped electron oscillators synchronize abruptly when the pump strength is increased through a sharp threshold and internal motions are radiatively cooled via coupling to a mode of a cold, microwave cavity. The collective motion is characterized by an instability which is approximated by a rigid model. Hysteresis is observed when either pump frequency or pump strength is swept. Time translation invariance requires that any coherent response be phase bistable, with the two degenerate phase states differing only in phase by 180 degrees. The collective behavior in this system is self-organized insofar as the choice between these bistable phase states is determined by internal motions. Parametrically-pumped electron oscillators are only partially synchronized, with a coherent component which is observed to be very sensitive to radiative cooling of the internal motions, providing a new technique for probing the standing-wave modes of a Penning trap cavity, *in situ* at 4K, without a microwave drive. Measured resonant frequencies of a specially designed cylindrical Penning trap agree very well with the eigenfrequencies of an ideal microwave cavity, typically to a percent or better. This cylindrical trap is of such high quality that one isolated electron in its cavity is observed with good signal-to-noise. For the first time, cavity modes of a Penning trap are clearly observed and identified with familiar field configurations. Among over 100 observed modes are some which are suited for cyclotron excitations, rapid change of cyclotron damping, sideband cooling of an electron to very low (mK) temperatures and directly driven spin flips. The extraordinary control over a wide range of parameters in this well-characterized system opens the way to a new generation of electron magnetic-moment measurements and other radiative studies, as well as experiments on collective behaviors and fluctuation phenomena.

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