Chapter 6

Conclusion

A new cylindrical Penning trap has been demonstrated to be a good approximation to an ideal microwave cavity. Anywhere from 1 to more than 10^5 electron oscillators can be isolated near the center of the trap cavity, localized in the simple standing wave patterns of the cavity modes. A single electron has been observed with good signal-to-noise ratio, demonstrating that precise measurements with one electron, such as measurements of the electron magnetic moment and of inhibited spontaneous emission, etc., can be performed in a cylindrical Penning trap, with the added advantages of a well-characterized microwave cavity.

Resonant cooling of electron oscillators by a cavity standing wave has made possible this first study of cooperative phenomena in parametrically-pumped electron oscillators. Phase bistable CM oscillations with long-term coherence emerge from weak, disordered fluctuations as the pump strength is increased through a sharp threshold, producing extraordinarily large coherent signals. Detailed experimental studies of the (h, ω_d) space of the parametric pump establish a hyperbolic region of instability, in which the quiescent state becomes unstable, with a threshold h_T which is proportional to the number of electrons and to the damping rate by the tuned circuit per electron. Analysis of a rigid model corroborates that this observed instability corresponds to the lowest order ($n = 1$) Mathieu instability in the presence of damping. Phase bistability and hysteresis are also observed and well approximated by an ordinary differential equation for rigid motions. Hysteresis in one parametrically-pumped electron oscillator may make it possible to detect relativistic cyclotron excitation [31] without perturbing the excitation during crucial stages.

A rigid model, however, is an oversimplification since internal motions and electron-cavity interaction are also important. Observed energy in the coherent CM motion is limited by cavity cooling of internal motions. For example, the slope of a linear lineshape (obtained when $|C_4|$ is large) increases as the internal motions are cooled. This extreme sensitivity to cavity cooling in partially synchronized motion provides a new technique for probing the radiation field modes of a Penning trap cavity, *in situ* at 4K, without a microwave drive. Measured eigenfrequencies agree well with those of an ideal cylindrical cavity. Interaction with a cavity standing wave is so well characterized that motional sidebands and splittings are observed in cavity mode resonances.

Furthermore, interesting fluctuation phenomena are observed in great detail. The center-of-mass of partially synchronized, parametric electron oscillators makes random transitions between degenerate phase states which differ by 180°, at a rate which increases with increasing internal energy. Detailed observations of phase jumps show a variety of "trajectories." A phase jump is often initiated by a collapse of one CM phase state to the "quiescent" state, followed by re-excitation to the other phase state. In some cases, increased fluctuation is observed as precursor to a phase jump. In a rare event, oscillations in the basins of attraction of the two phase states have also been seen.

Although some insight has been gained, many interesting questions remain to be explored. Are there other collective states which are excited by the parametric pump? Why does observed coherent CM motion above threshold saturate? Is there a simple explanation for the convergence of power in the peak and fluctuation pedestal of the response spectrum? Above threshold, how are the amplitude and phase fluctuations related, and how do they vary with system parameters? As shown in this work, simple notions of synchronization, a rigid model, and electron-cavity interaction have been useful but are inadequate for further investigation.

Unfortunately, a better treatment is not available. The underlying simplicity suggested by our observations of Lorentzian lineshapes of cavity mode resonances, an exponential distribution of residence times, etc. will hopefully prompt a detailed theoretical analysis of parametrically-pumped electron oscillators. Below threshold, perhaps a simple, generalized set of rate equations can be obtained for the energy transfer processes within the cryogenic micro-plasma [68,67].

As an initial application, partially synchronized electrons are used to identify the radiation modes of a trap cavity for the first time. A thousand-fold decrease in an electron's axial temperature now seems feasible, as does a new generation of electron magnetic moment measurements which avoid previous limitations from damping linewidth and cavity shifts of measured frequencies. Extraordinary control over a wide range of parameters in this well-characterized system provides an ideal environment for such precision measurements, and for new experiments in nonlinear dynamics with few electrons, cooperative behavior in increasingly larger systems, radiative effects due to strong coupling of many electrons localized within a fraction of a wavelength of a cavity standing wave, and fluctuation phenomena.