Chapter 1

Introduction

Isolated elementary particles and ions in electromagnetic traps have made possible a growing variety of experiments, ranging from the most stringent tests of renormalized QED with the measured electron magnetic moment [88] and tests of CPT in the proton/antiproton system [36], to trapping large numbers of cryogenic positrons and anti-protons for synthesis of anti-hydrogen, to studies of pure electron plasmas [61,68] and spatially ordered structures in laser-cooled, trapped ions [21,95,47,40,16]. This work presents the first study of recently discovered [78,79] self-organized [65] collective behaviors in a system of cavity-cooled, parametrically-pumped electron oscillators which are isolated in a new cylindrical Penning trap cavity.

1.1 Cooperative Phenomena

Centuries ago, Huygens observed that pendula of two clocks on a wall tend to synchronize [81]. More recent efforts to characterize large dynamical systems have generated increased interest in larger systems of coupled oscillators. Extensive studies of cooperative behavior in the laser, for example, revealed strong similarity in this nonequilibrium system to critical phenomena in a ferromagnet, developing concepts and techniques analogous to those of phase transition theory. [42,17] Also, arrays of Josephson junction oscillators synchronize when they produce high frequency microwaves, being coupled via a common load of passive circuit elements

[43,60]. Large systems of well-characterized, coupled oscillators are difficult to realize under good control in the laboratory and no unified theoretical approach has yet emerged for collective behavior far from thermal equilibrium. Nonetheless, an increasing number of works studying a few simple systems of coupled limit cycle oscillators are revealing recognizable cooperative phenomena such as oscillator synchronization, "clustering" and "attractor crowding" [91,25]. Examples include Van der Pol oscillators [96] and an "active rotator" model [58] which are studied using various techniques: solutions of coupled differential equations [96,66,62], coupled iterative maps [74,91,25], generalized mean field approaches [58], as well as renormalization-group analysis [15].

Parametrically-pumped electron oscillators are far from thermal equilibrium insofar as they are strongly driven and they continuously dissipate energy. A unique feature is that the oscillators synchronize to produce an observable, coherent motion of their center-of-mass (CM) at half the frequency of the pump. Time translation symmetry requires that any such response be bistable in phase relative to a subharmonic of the parametric pump. The collective motion is self-organized insofar as the choice between the bistable phases depends upon the internal motions of the electrons (not upon the external pumping field) and characteristically requires sufficient energy dissipation [65]. Transitions between the bistable phase states depend upon the internal energy of the oscillators (relative to their CM), reminiscent of a two state system coupled to a thermal bath. This energy is varied by tuning the radiative dissipation to the cold microwave cavity formed by electrodes of a specially designed, cylindrical Penning trap. The collective motion is characterized by an instability which is well approximated by a rigid model, with hysteresis occurring when either pump frequency or pump strength is swept back and forth through a region of instability. Partial synchronization is manifested by interesting effects, such as the broadening of fluctuation spectra, saturation of the coherent component as pump power is increased above a threshold, slow relaxation to steady level in pulsed excitation, etc..

Likely applications of the newly developed techniques include a thousand-fold reduction in the axial temperature of a trapped elementary particle, radiative cooling of cryogenic electron plasmas and internal motions of molecular ions at adjustable rates, and a new generation of measurements of the electron magnetic moment which are no longer limited by cavity frequency shifts or by damping linewidths.

1.2 Eliminating Cavity Shifts

Although the observed nonlinear dynamics and collective behaviors in parametrically-pumped electron oscillators are of interest in themselves, the development of a high quality cylindrical Penning trap which has made this study possible was originally motivated by the goal of starting a new generation of electron magnetic moment measurements in well-characterized radiation fields. The discovery of synchronized motions in parametrically-pumped oscillators has allowed us, for the first time, to clearly observe and identify microwave standing wave modes in a trap cavity, greatly accelerating progress towards this goal.

Measurements of the anomalous magnetic moment of the electron a provide the most stringent test of quantum electrodynamics (QED) [52]. This theory predicts corrections to the simplest Dirac theory due to the interaction of an electron with the fluctuating radiation modes of the electromagnetic vacuum. It relates a to an asymptotic series in powers of the fine structure constant α,

$$a = C_1\left(\frac{\alpha}{\pi}\right) + C_2\left(\frac{\alpha}{\pi}\right)^2 + C_3\left(\frac{\alpha}{\pi}\right)^3 + C_4\left(\frac{\alpha}{\pi}\right)^4 + \dots \quad . \tag{1.1}$$

Over four decades, measurements of a [14,88,89] and α were greatly improved, as were QED calculations [52,53] of the expansion coefficients C_i. The highest accuracy measurements [88,89] of a employ a single electron in a Penning trap [10] to obtain an accuracy $\Delta a/a < 4 \times 10^{-9}$. This unrivaled comparison of a measured and calculated property of an elementary particle, reveals an agreement $\Delta a/a < 4 \times 10^{-8}$ which would have astounded those who were struggling to formulate renormalized QED.

A few years ago, experimental progress in measuring the anomalous magnetic moment a was seriously interrupted. The electromagnetic vacuum in which the electron was located was discovered to be significantly modified by the metal electrodes of the Penning trap [32]. Electron cyclotron motion around a vertical magnetic field $B = 6$ Tesla is at frequency

$$\omega_c = \frac{eB}{mc} = 2\pi\ (164\ \text{GHz}). \tag{1.2}$$

Cyclotron motion in free space would would damp via synchrotron radiation at a rate

$$\gamma_c = \frac{4e^2\omega_c^2}{3mc^3} = (0.1\ \text{sec})^{-1}. \tag{1.3}$$

Instead, the decay of cyclotron energy [32] for a single electron in the trap was shown to be decidedly less by a factor of 3, the first observation of inhibited spontaneous emission within a microwave cavity [22]. Corresponding cavity shifts of measured frequencies, calculated [8,9,12] but not yet observed, were estimated to be the largest experimental uncertainty [88,89] based upon the calculations. To complicate this serious problem, so little was known about the radiation field within trap cavities that the uncertainty estimate is itself rather suspect. Also, the traditional hyperbolic electrode geometries do not allow easy calculation [12] or even a ready classification of the standing-wave fields in cavity radiation modes. Even if mode eigenfrequencies were known, the field configuration and hence the coupling of a centered electron to any particular cavity mode (if any) would not be known.

Although the importance of cavity shifts for measurements of the electron magnetic moment was demonstrated only recently, the basic notion that the couplings of two oscillators can shift both the damping rate and oscillation frequency of the oscillations (Fig. 1.1) is certainly very familiar. (The electron cyclotron motion and an electromagnetic cavity mode are the coupled oscillators here.) Long ago, for example, it was mentioned that the spontaneous emission of an atom placed in a cavity could be inhibited [70]. Further discussions of cavity-induced modifications to atom damping rates came later [55], with clear realization of the problems

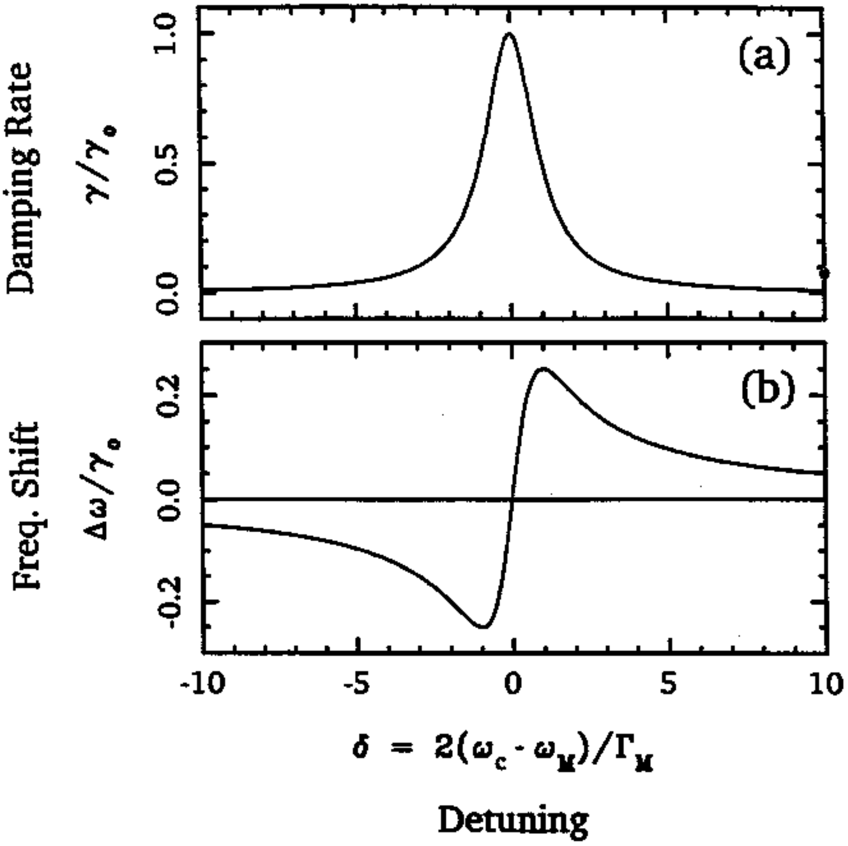

Figure 1.1: Characteristic dependence of an oscillator's damping rate γ in (a) and frequency shift $\Delta\omega$ in (b), as a function of its detuning δ from the resonant frequency of a coupled cavity mode (or LCr circuit).

that the frequency shifts would present for precise measurements of resonance frequencies [54]. In fact, soon after the observation of inhibited spontaneous emission in a trap cavity, similar effects were observed with Rydberg atoms traveling between parallel conducting plates [48] and in another Penning trap [88,89]. Related studies with Rydberg atoms continue [44,46]. Some additional evidence for the presence of cavity modes in a hyperbolic trap has also been observed [87] but remains difficult to interpret since signal-to-noise was poor, no Lorentzian lineshapes were established and no information about the standing wave field configurations (and hence the coupling to a trapped electron) could be deduced.

The new experiments made possible by the synchronized motions of parametrically-pumped electron oscillators in a high quality cylindrical trap and described here show how to change the cavity-modified vacuum from a serious interruption into an advantage. The well-characterized, standing wave fields of the cylindrical trap cavity revives interest in cavity-shifts of an electron's spin precession frequency. Theoretical studies first suggested that the cavity modified vacuum could be responsible for shifts large enough to be observable [26]. Brown and Boulware [4] contradicted the initial claim. Many other theoretical papers were written [27,5,76,56,80,57,2]. The latest work seems to support the contradiction of the initial claims, even though opposing conclusions have never been resolved as completely as might be desired. The theoretical studies share the common difficulty of making a calculable model (eg. a spin near a conducting plate or plates) which is also a reasonable approximation to an electron in a trap cavity. It remains to theoretically study the resonant interaction of a spin with one of the high Q modes of the cylindrical trap cavity which couple most strongly to a spin. If it is experimentally demonstrated that cavity shifts of the cyclotron frequency are well understood, cavity shifts of the spin frequency could then be investigated experimentally as well.