A Superconducting Solenoid System Which Cancels Fluctuations in the Ambient Magnetic Field

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An extra superconducting coil added to a standard, high-field solenoid results in a self-shielding solenoid system which utilizes flux conservation to passively shield an interior volume from changes in the ambient field, such as those from elevators or subways. For this first experimental demonstration, a highly homogeneous 6 T solenoid and an added coil were arranged in one of the geometries predicted to produce effective shielding. The fluctuations in the shielded high-field region are observed to be smaller than the fluctuations in the spatially uniform ambient magnetic field by a large factor of 156, confirming the general shielding principles presented earlier. This shielding is crucial for ongoing antiproton cyclotron resonance experiments and should be useful for nuclear magnetic resonance experiments and ion cyclotron resonance experiments and for other applications where high field and high stability are required simultaneously.

In ion cyclotron resonance and nuclear magnetic resonance experiments, the cyclotron frequencies of ions and the precession frequencies of nuclear spins in a strong magnetic field are precisely measured and compared to the frequencies of other species in the same strong field. Since these frequencies are proportional to the magnetic field, the accuracy is compromised when the magnetic field changes during measurements. While high-frequency fluctuations in the ambient magnetic field are shielded by eddy currents induced in cylindrical conductors between the solenoid and the experiment (e.g., a metal support for the solenoid and dewar walls), low-frequency fluctuations in the ambient field generally cause significant changes in the magnetic field and the measured frequencies. The passive shielding experimentally demonstrated here for the first time greatly reduces the effect of the field fluctuations, confirming an earlier theoretical prediction (*1*). As an example, an antiproton ICR measurement with a fractional accuracy of 4×10^{-8} was recently done in a 6 T superconducting solenoid located near a large-particle accelerator (*2*). Without the shielding, the magnetic fluctuations in this environment (discussed below) would have limited the measurement accuracy to 1 ppm. Shielding the fluctuations in the ambient field is crucial for achieving the highest accuracy measurements (*3, 4*). However, even for less precise experiments, good shielding makes it possible to make measurements much nearer to sources of fluctuating magnetic fields.

Subways, nearby elevators, and cars pulling into nearby parking lots often produce the largest variations in the ambient field. Even when these man-made sources are

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absent, the earth's field can vary. Depending upon location, variations of 10 nT (100 μG) to 100 nT (1 mG) are observed, and larger variations are possible during magnetic storms which are related to solar activity (*5*). Figure 1 shows the typical situation in our laboratory as measured with a fluxgate magnetometer. During a window of a few hours at night (Fig. 1a), when the subway is not running, the fluctuations are of order 1 nT (100 μG) with occasional steps of order 60 nT (600 μG). By day (Fig. 1b), much larger fluctuations up to 300 nT (3 mG) are typical. Simultaneous measurements of the fluctuating ambient field, with probes separated by several meters, showed that the fluctuating ambient field is typically spatially uniform. Figure 2 shows an example of field variations in the more hostile environment of an accelerator laboratory (at CERN in Geneva, Switzerland) at the location of the antiproton ICR measurement mentioned (*2*). The large 4 μT (40 mG) peaks every few seconds are due to the nearby proton synchrotron (PS). This accelerator ring is 300 m in circumference and we are located outside the ring, about 19 m away. The smaller steps of order 1 μT (10 mG) are due to the low-energy antiproton ring (LEAR), a small accelerator ring which is 80 m in circumference and located a few meters away from the antiproton experiment. Nearby elevators, moving cars, etc., typically make fluctuations in the ambient field which are somewhere between these two examples. We found, for example, that opening

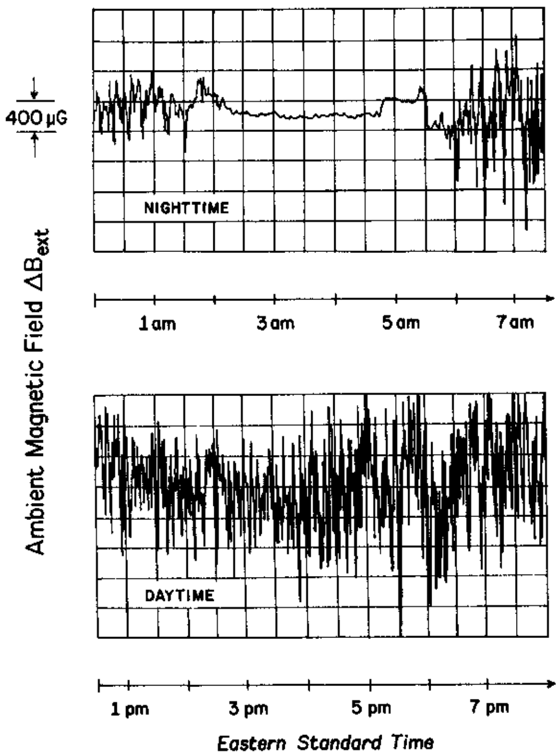

FIG. 1. Typical fluctuations in the magnetic field in our laboratory as measured using a fluxgate magnetometer and a detection bandwidth of 0.01 Hz. The quiet window during the night occurs when the subway is shut down.

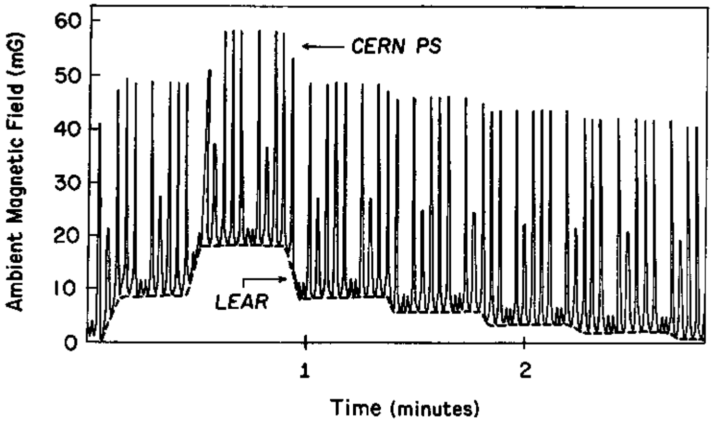

FIG. 2. Magnetic field fluctuations at the location of an antiproton ICR experiment at the CERN accelerator facility. The largest variations every 2.4 s are from the nearby CERN proton synchrotron. These are superimposed on the slower field shifts produced by the LEAR storage ring during a deceleration cycle.

a metal privacy door in an adjacent bathroom changed the magnetic field in one of our laboratories by 50 nT (500 μG).

Unfortunately, the familiar techniques to shield low-field regions by using highly permeable shielding materials do not work in high-field regions. Highly permeable materials saturate and provide no shielding when saturated. To be useful, permeable metal shields must thus be extremely large so that they can be located far from the solenoid. Shields made of type I superconducting materials like lead and niobium are also not effective in large fields (e.g., 6 T) because such fields are above the critical field for these materials. An attempt has been made to use a type II superconducting shield to shield external fluctuations from a very small high-field region (*6*), but there was some trouble with flux jumps associated with the shield and in general the spatial homogeneity of the magnetic field would be compromised for small shields.

In the alternate approach reported here, the geometry of the superconducting solenoids that produce the large magnetic field is chosen so that external fluctuations are canceled at the location of the experiment by extra currents induced in the solenoids. A family of appropriate geometries is described by a precise mathematical condition (*1*). The particular geometry tested here is one of several that is especially suited to high-resolution ICR and NMR experiments. An additional superconducting coil, with its ends connected, is incorporated into an otherwise standard NMR solenoid system designed to produce a highly homogeneous 6 T field. In this first trial, magnetic field fluctuations in the central volume where experiments are located are reduced by a very large factor of 156 (compared to the ambient field fluctuations) and the high degree of spatial homogeneity within the NMR solenoid is not compromised. Moreover, the shielding is entirely passive, utilizing flux conservation in coupled superconducting solenoids, and requires no adjustment or electronics. This demonstration shows that significant shielding can be realized with modest additions to existing designs for commercially available, high-field solenoid systems, if the additions are made in accord with theoretical design principles presented earlier (*1*).

Finally, we note that for typical NMR and ICR experiments, the coupled solenoids do not and need not shield the components of the fluctuating, ambient field which are perpendicular to the strong field B_0. A fluctuating transverse field $B_\perp$ provides only a quadratic correction to the magnitude of the strong field:

$$B = B_0\sqrt{1 + \left(\frac{B_\perp}{B_0}\right)^2}\,. \quad [1]$$

Even an extremely large transverse field $B_\perp = 6\ \mu\mathrm{T}$ (60 mG) thus results in an extremely small fractional change in the field, $\Delta B/B < 10^{-14}$.

DEFINITIONS AND ILLUSTRATION

To illustrate the basic shielding scheme, consider an axially symmetric solenoid i made of superconducting wire with ends connected to make a closed circuit. The potential difference around the shorted solenoid is zero. By Faraday's law, an externally applied field B_e induces a current I_i in the solenoid which in turn produces an induced magnetic field B_i sufficient to keep the flux through the solenoid from changing:

$$\int_i [B_e + B_i]dA = 0. \quad [2]$$

We take the conserved value of the flux to be 0 so that we can focus on fluctuations from some steady state. The subscript on the integral indicates integration over the area of solenoid i. The induced current persists since the resistance around the superconducting circuit is zero.

In what follows, we use cylindrical coordinates ρ and z, so that $B_i = B_i(\rho, z)$, for example. The net field at the center of the solenoid $B_e(0, 0) + B_i(0, 0)$ can be written in terms of the shielding factor S as $B_e(0, 0)/S$. Perfect shielding corresponds to $S \rightarrow \infty$ and $S^{-1} = 0$, since

$$S^{-1} = 1 + \frac{B_i(0, 0)}{B_e(0, 0)}\,. \quad [3]$$

In light of the flux conservation criterion, Eq. [2], this can be written as

$$S^{-1} = 1 - \frac{\int_i B_e dA/B_e(0, 0)}{\int_i B_i(\rho, z)dA/B_i(0, 0)}\,. \quad [4]$$

To aid intuitive interpretation, we note that S^{-1} is linearly dependent on the ratio of two averaged and normalized fields

$$S^{-1} = 1 - \frac{\overline{b_e}}{\overline{b_i}} \quad [5]$$

defined by

$$\overline{b_e} = \frac{\int_i B_e(\rho, z)dA}{B_e(0, 0)\int_i dA} \quad [6]$$

$$\overline{b_i} = \frac{\int_i B_i(\rho, z)dA}{B_i(0, 0) \int_i dA}. \qquad [7]$$

Here $\int_i dA$ is the total area involved in the flux integration for circuit i. Perfect shielding requires a solenoid for which the normalized average values of the external field and the induced solenoid field are equal, $\overline{b_e} = \overline{b_i}$.

Without explicit calculation, one can immediately see that complete shielding of spatially uniform fields is possible with a single superconducting solenoid circuit, even if the solenoid has many layers of windings. For a spatially uniform external field B_e we have $\overline{b_e} = 1$ and the shielding is given by

$$S^{-1} = 1 - \frac{1}{\overline{b_i}}. \qquad [8]$$

We now consider two limiting cases. For a short solenoid (e.g., a loop), the induced field near the windings is larger than the induced field near the center. The normalized average value in the bore $\overline{b_i}$ is thus greater than 1 so that S^{-1} is positive. On the other hand, a long solenoid produces a uniform field within, decreasing slightly at the ends where the field lines separate. The normalized average of the induced field within the solenoid is thus slightly less than the field at the center so that $\overline{b_i}$ increases to a value of 1 with increasing length. Consequently, S^{-1} is negative, increasing with solenoid length to a limit of 0. Since S^{-1} must cross 0 between these two limiting cases, complete shielding is obtained with an appropriate choice of dimensions.

COUPLED SUPERCONDUCTING SOLENOIDS

A single solenoid which is capable of producing a high magnetic field while also providing shielding would generally have impractical dimensions and would not produce a spatially homogeneous magnetic field (*1*). Practical solenoid systems typically contain several persistent superconducting solenoids, one to produce the large field and the others as shims to make the field near the center as homogeneous as possible. We therefore consider a system of $i = 1, \ldots, N$ persistent superconducting solenoids, each of which is axially symmetric. A current I_i in circuit i produces the field $B_i(\rho, z)$. If the currents are represented by a column vector **I**, components of a column vector **g** relate the field at the center to the current which generates it:

$$B_i(0, 0) = g_i I_i. \qquad [9]$$

The areas of the circuits are represented by the column vector **A** with components

$$A_i = \int_i dA, \qquad [10]$$

which can be easily generalized for the case of a nonuniform field (*1*). The familiar symmetric inductance matrix **L** has components given by

$$\int_i B_j(\rho, z)dA = L_{ij} I_j. \qquad [11]$$

A diagonal element L_{ii} is the self-inductance associated with circuit i and off-diagonal elements are the mutual inductances between circuits. The shielding factor is (*1*)

$$S^{-1} = 1 - \mathbf{g}^{\mathrm{T}}\mathbf{L}^{-1}\mathbf{A}, \qquad [12]$$

with the superscript T indicating transposition so that $\mathbf{g}^{\mathrm{T}}$ is a row vector. Complete shielding occurs when $S \rightarrow \infty$, in which case

$$\mathbf{g}^{\mathrm{T}}\mathbf{L}^{-1}\mathbf{A} = 1. \qquad [13]$$

This is the precise mathematical condition for a self-shielding solenoid system. For a particular application, the goal is to find a practical geometry which retains other desired properties of the magnetic field that is produced (such as spatial homogeneity) and also satisfies this condition as accurately as possible.

The specific design of a self-shielding solenoid requires calculating S^{-1} from Eq. [12] as a function of the solenoid dimensions. The dimensions are varied until $S^{-1} = 0$, whereupon Eq. [13] is satisfied. Simple geometries and dimensions which seemed theoretically promising were described earlier (*1*). However, high-field NMR solenoids with state-of-the-art spatial homogeneity are generally constructed commercially, with complicated geometries which vary from manufacturer to manufacturer. For this first demonstration, Nalorac Cryogenics provided us with the (proprietary) internal dimensions of their standard, high-homogeneity solenoid made for NMR applications. Our detailed calculations for finding a practical geometry which satisfies the shielding condition in Eq. [13] indicated that a small, persistent, interior solenoid could be added to make a solenoid system which cancels external fluctuations. No other modification of the standard system was needed to preserve the high degree of spatial homogeneity ($\Delta B/B < 10^{-8}$ over a sphere 1 cm in diameter). A cross section of the solenoid and the added interior coil (to scale) is shown in Fig. 3. The strong field produced within the solenoid is along the symmetry axis of the solenoid. The large outer coil produces the large magnetic field and contains shaped correction coils to obtain a high degree of spatial homogeneity. Without the added interior solenoid, this cross section is typical of a high-field NMR solenoid.

MEASURED SHIELDING

To measure the shielding, we insert an NMR probe with an acetone sample into the high-field region. The sample is a sphere with a diameter of 1 cm. We apply an external magnetic field to the solenoid using large square Helmholtz coils, which are 2.81 m on a side and are separated by 1.53 m. These coils produce a magnetic field which varies over the solenoid by less than 0.3%.

By opening the circuit of the inner shielding coil, we measure a shielding which is typical of a high-homogeneity, high-field NMR solenoid. Figure 4a shows the field change within the central volume of the solenoid (from the measured shift in NMR frequency) as a function of the external field applied with the Helmholtz coils. The measured shielding factor is

$$S = -4.27 \pm 0.07. \qquad [14]$$

This means that external field fluctuations are reduced by this factor, and we expect

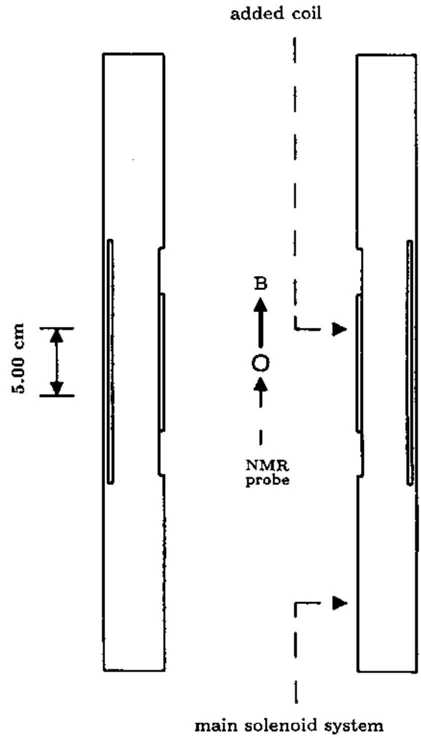

FIG. 3. Cross section of the windings of the high-field solenoid which produces a vertical magnetic field. The innermost solenoid was added to make the solenoid system cancel external fluctuations.

that this number is rather typical of high-field solenoids of reasonable geometry since it is rather insensitive to the details of the geometry. The negative sign is also typical. It indicates that the solenoid overcompensates the external fluctuation, so that the fluctuation experienced in the center region is actually oppositely directed to the applied external field. The measured value compares well with the previously calculated value $S = -4.5 \pm 0.1$. (The uncertainty in the calculation reflects the inaccuracy in our inductance calculation and includes a crude estimate of the effect of changing the size of the solenoid system by an amount corresponding to the imprecision in our knowledge of the location of the windings.)

When the inner solenoid is allowed to go persistent, the shielding improves dramatically, as indicated in Fig. 4b. The measured shielding factor is

$$S = -156 \pm 6. \quad [15]$$

We interpret this as the shielding for a spatially uniform field, after increasing the uncertainty from 3 (the measurement precision) to 6 to include effects of possible inhomogeneities in our applied field. We observe the same linewidth in the NMR signal with the shielding coil as without, indicating that the spatial homogeneity is not compromised over the 1 cm diameter of the spherical NMR probe.

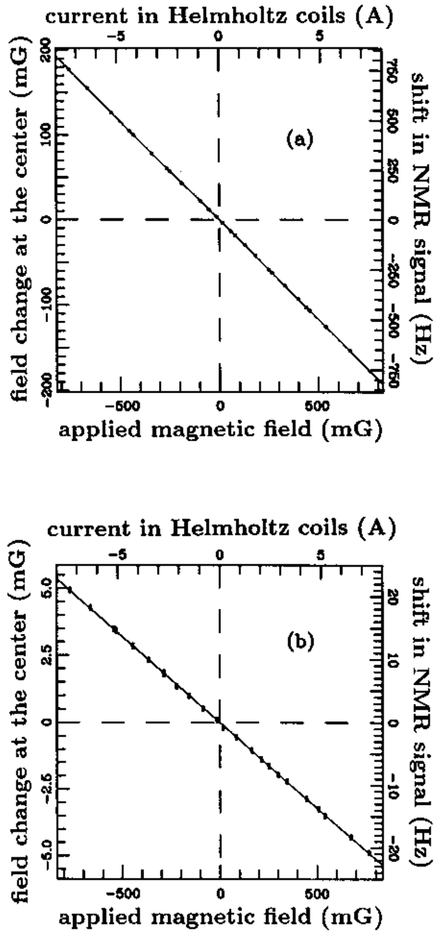

FIG. 4. Change in the magnetic field measured within the solenoid (deduced from the change in NMR frequency of an acetone sample) as a function of the external magnetic field applied over the solenoid, without (a) and with (b) the added inner solenoid.

SHIELDING OF NONHOMOGENEOUS FIELDS

Most uncontrollable sources of magnetic field fluctuations are located some distance away from the solenoid. As a result, the fluctuating field tends to be spatially uniform across the solenoid and the tested solenoid system was specifically designed to shield fluctuations in the spatially uniform ambient field. Moreover, a linear gradient in the applied field at the solenoid averages to zero over the solenoid, thus making no contribution to $\overline{b_e}$ and to the shielding factor.

For nearby sources, however, or when nearby magnetic materials distort the fluctuating applied field, the shielding can be modified significantly. Great care must thus be taken to apply a relatively uniform field when the shielding is to be measured. We have observed greatly exaggerated shielding factors by applying an external field with a single loop around the solenoid, for example, or by using Helmholtz coils which are too small. As another example, steel reinforcements in concrete blocks located beside the solenoid at CERN distort the fluctuating field from the nearby storage rings. For the CERN PS, we observe a shielding factor of $S = -110$. The field from the closer LEAR magnets is shielded by a factor of $S = -50$. These shielding factors are significantly lower than the shielding factor of $S = -156$ observed for uniform magnetic fields, but are still very large reductions in the fluctuations of the magnetic field in the volume within the solenoid where experiments are located.

SUMMARY

In principle, a system of superconducting solenoids can be configured to produce a strong magnetic field while also exactly canceling any fluctuation in the ambient magnetic field (*1*). Any of a variety of geometries satisfies a precise mathematical condition which describes perfect shielding, making it possible to design superconducting solenoids which cancel the effect of spatially uniform, ambient field fluctuations while also preserving other important magnetic field properties such as spatial homogeneity.

The measurements reported here demonstrate for the first time that it is practical to realize significant shielding in otherwise standard superconducting solenoids constructed for NMR and ICR measurements. The most common and important fluctuations in the ambient magnetic field are spatially uniform over the solenoid, and such fluctuations are canceled within the shielding solenoid by a factor of 156, making possible much more accurate ICR and NMR measurements. Even when highest precisions are not required, the passive shielding makes it possible to locate experiments nearer to sources of magnetic field fluctuations such as elevators, parking lots, and subways.

In the future, even larger shielding factors may be realized by adding small coils to the outside of a superconducting solenoid system (*1*), since these could be adjusted during solenoid construction to optimize the shielding. Moreover, shielding could be realized in rather different geometries for other applications, provided that the same design principles are followed.

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