One electron in an orthogonalized cylindrical Penning trap

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Highest precision mass spectroscopy and other highly accurate measurements have been carried out in Penning traps with metal electrodes painstakingly shaped along hyperbolic contours. A single electron has now been observed in a much simpler, cylindrical trap with equally good signal-to-noise ratio, thus demonstrating the possibility of conducting such experiments in a more readily constructed apparatus and opening the possibility of some new experiments. An essential requirement is the careful choice of electrode lengths to make the size of the electric quadrupole potential insensitive to adjustments which minimize deviations from an electric quadrupole.

Penning traps are an important tool for precision tests of fundamental physics, high-resolution mass spectroscopy, studies of single elementary particles, and radiative studies.[1] A key feature is the high accuracy with which the oscillation frequencies of charged particles within these traps can be measured. Specific examples include the measurements of the anomalous magnetic moments (g-2) of the electron and positron,[2] the proton/electron mass ratio,[3] observation of relativistic hysteresis and bistability of a single electron (stored longer than ten months),[4] and inhibition of spontaneous emission.[5] An ideal Penning trap for precise measurements consists of a spatially uniform magnetic field which is constant in time[6] and a pure electrostatic quadrupole potential. So far, the experiments employ metal electrodes painstakingly constructed along hyperbolic surfaces of revolution which are equipotentials of the desired quadrupole potential. There are nonetheless significant deviations from the electric quadrupole because of electrode imperfections and misalignment, along with holes and slits[1] for particle loading and detection. These deviations generate anharmonic particle oscillations. Hence, the highest precision experiments are impossible, due to the resulting amplitude-dependent frequency shifts and linewidth broadening, until the potential on a set of compensation electrodes is adjusted.[7] Such adjustments are generally attended by very inconvenient shifts in the resonant frequency of oscillation when the particles are driven along the magnetic field axis.

Theoretical studies[8,9] of the electrostatic properties of hyperbolic traps showed that the undesired frequency shifts can be eliminated by a judicious choice of an "orthogonalized" trap geometry. This geometry is now being incorporated into a second generation of compensated hyperbolic traps.[10] The calculations also suggested that the hyperbolic electrodes may not be as essential as had been assumed, leading to consideration of cylindrical trap geometries as an alternative.[11] Cylindrical electrodes can be machined to higher accuracy in less time and the electrostatic properties can be calculated analytically, greatly facilitating new designs for particular applications. For example, open-endcap Penning traps[12] composed only of cylindrical rings make it possible to load and cool antiprotons[13] for comparison of their inertial mass to that of protons. In general, traps with cylindrical electrodes will produce undesired frequency shifts which are extremely large, much larger than for the first generation of compensated hyperbolic traps. We demonstrate here that a carefully chosen cylindrical geometry can be utilized even for the precise measurements mentioned. A single electron is observed with a signal-to-noise ratio which compares favorably with that obtained in hyperbolic traps. In fact, the frequency resolution now seems limited by thermoelectric effects, not by the quality of the electric quadrupole.

Figure 1 shows the trap geometry selected for these studies. Potentials $+1/2\,V_0$ on the flat endplates and $-1/2\,V_0$ on the center ring produce the quadrupole potential. A potential V_c on the pair of compensation rings is adjusted to minimize deviations from the electric quadrupole. The height of these electrodes,

$$\Delta z_c/z_0 = 0.20, \qquad (1)$$

is sufficient to avoid an unacceptable sensitivity to the mechanical tolerances. A corresponding choice of radius,

$$\rho_0 = z_0 = 1.186, \qquad (2)$$

was calculated to minimize the undesired frequency shifts which have been mentioned.[11] We find it convenient to specify the overall size of the trap with a characteristic length $d = 0.355$ cm, where d is defined by

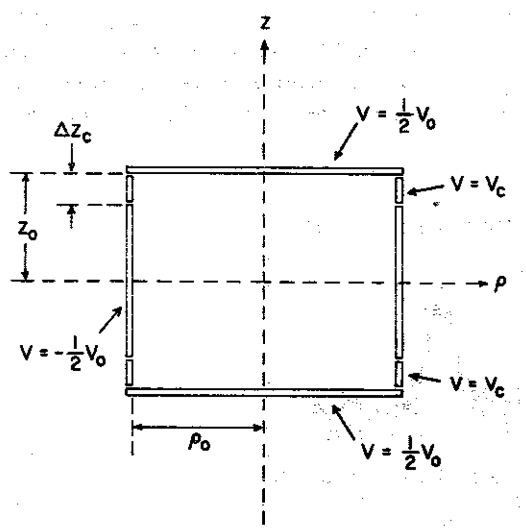

FIG. 1. Orthogonalized cylindrical trap to scale with $z_0 = 0.385$ cm. The z axis is aligned with a spatially uniform 5.6 T magnetic field.

$$d^2 = \frac{1}{2}(z_0^2 + \frac{1}{2}\rho_0^2) \tag{3}$$

and $z_0 = 0.385$ cm. Gaps between the electrodes (so different potentials can be applied to different electrodes) are made as small as practicable at 0.013 cm. The small gaps are not expected to have major consequences.[12] The trap is located in a high-vacuum envelope which is cryopumped because the envelope is kept at 4.2 K via a thermal contact with a liquid-helium bath.

To summarize the properties of the orthogonalized cylindrical trap, we use an expansion of the potential near the trap center:

$$V(\mathbf{r}) = V_0\left(\frac{z^2 - (1/2)\rho^2}{2d^2}\right) + \frac{1}{2}V_0 \sum_{\substack{n=0 \\ \text{even}}}^{\infty} C_k\left(\frac{r}{d}\right)^k P_k(\cos\theta), \tag{4}$$

where d is the characteristic length for this trap and the sum is over even k because of symmetry under reflections $z \to -z$. The first term on the right is the desired electric quadrupole potential in cylindrical coordinates (ρ,z). The second term represents undesired additions in an imperfect trap, expanded in spherical coordinates (r,θ) and Legendre polynomials $P_k(x)$. The expansion coefficients are linear in the compensation potential V_c:

$$C_k = C_k^{(0)} + D_k V_c/V_0, \tag{5}$$

with $C_k^{(0)}$ and D_k exactly calculable[11] using standard techniques[14] for perfectly aligned and cleaned conducting electrodes. The expansion converges rapidly for particles near the center of the trap where $(r/d) \leqslant 10^{-2}$, so we only consider lowest order coefficients C_2 and C_4 (C_0 having no observable consequences for particles confined in the traps).

The compensation potential V_c is tuned to make $C_4 = 0$, thereby eliminating the leading deviation from a quadrupole potential. This occurs when $V_c/V_0 = -C_4^{(0)}/D_4$ from Eq. (5). In practice, we tune V_c while monitoring the coherent response of a single trapped electron whose oscillatory axial motion along the magnetic field axis is being driven. For a trapping potential $V_0 = -10.193(1)$ V, we tune to $V_c = 3.574(1)$ V to get a symmetric resonance line shape as shown in Figs. 2 and 3(b). Thus, we get an experimental value of $V_c/V_0 = -0.35$ in good agreement with calculated value $V_c/V_0 = -0.34(2)$, the uncertainty being due to the estimated mechanical tolerance of 8×10^{-4} cm in the dimensions. The good agreement verifies the calculated value of $D_4 = -0.066(1)$. Detuning V_c to either side of this optimal value produces the characteristic skewed line shape of an anharmonic oscillator. This is illustrated in Figs. 3(a) and 3(c), where V_c is $+20$ and -20 mV from the optimum, respectively. Since the compensation voltage must be tuned to within 1 mV of the optimal value to avoid an observable skewing of the resonance line shape, we use the calculated value of D_4 to conclude that we are able to tune $|C_4| < 10^{-5}$ in this trap. With optimal tuning, the driven axial resonance is only 4 Hz wide (Fig. 2). The signal-to-noise ratio allows a shift in the axial frequency of 1 Hz out of 64 MHz to be easily resolved, a resolution which is comparable with that obtained in hyperbolic traps.

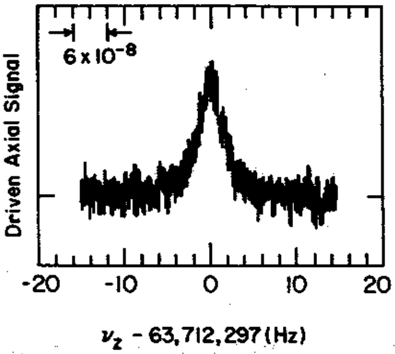

FIG. 2. Driven axial resonance of one electron in a cylindrical Penning trap.

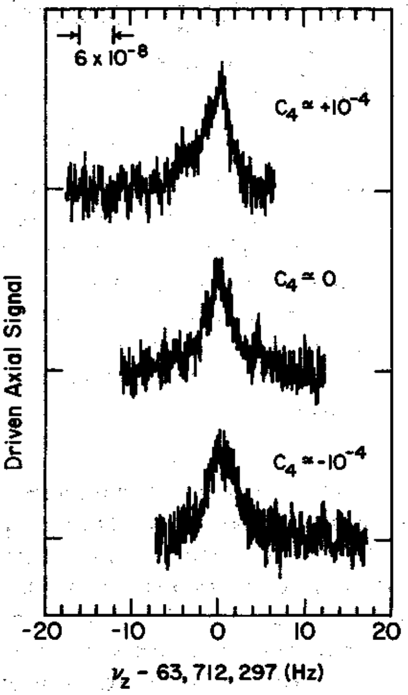

FIG. 3. Driven axial resonance for one electron in the orthogonalized cylindrical trap. The middle curve (b) is the symmetric line shape for a well-tuned trap ($C_4 \approx 0$). The characteristic line shapes for an anharmonic oscillator are obtained when the compensation potential is adjusted by $+20$ mV so that $C_4 > 0$ in (a) and by -20 mV so that $C_4 < 0$ in (c).

In general the axial resonance frequency ω_z will be shifted when V_c is changed since

$$\omega_z^2 = (eV_0/md^2)[1 + C_2^{(0)} + D_2 V_c/V_0]. \tag{6}$$

for the oscillation of a particle with charge e and mass m. The ideal electrode proportions were chosen to make $D_2 = 0$, so adjustments in the compensation potential leave ω_z unchanged. This is the condition for an orthogonalized, compensated Penning trap. Differentiating Eq. (6) and neglecting the small $C_2^{(0)}$ we get

$$D_2 \approx \left(\frac{\partial\omega_z}{\partial V_c}\right) \Big/ \left(\frac{\partial\omega_z}{\partial V_0}\right). \tag{7}$$

Using Eq. (7), we measure $D_2 \approx 2 \times 10^{-5}$ for this cylindrical trap, which is substantially smaller in magnitude than $D_2 \approx -5 \times 10^{-3}$ for the first generation hyperbolic traps used in so many precision experiments. Figure 3 illustrates how little ω_z shifts for a substantial adjustment of V_c to either side of the optimal tuning point, where the anharmonic distortions are clearly visible. Comparable changes in the early hyperbolic traps would be accompanied by a frequency shift of about 500 linewidths.

A measure of the quality of an orthogonalized trap is how little the axial frequency ω_z changes for a given change in C_4. To quantify this a quality factor

$$\gamma \equiv D_2/D_4 \tag{8}$$

has been defined with $\gamma = 0$ representing a perfectly orthogonalized trap. Using the measured value of D_2 and the calculated value of D_4, we obtain $\gamma \simeq 3\times10^{-4}$ which is a substantial improvement over $\gamma \simeq 0.56$ for the first generation hyperbolic traps.

The potential within the trap when $1/2\, V_A$ and $-1/2\, V_A$ are applied to the upper and lower endcaps, with all the other electrodes grounded, is important for damping, driving, and shifting the center of oscillation for the axial motion of trapped particles.[9] Near the center

$$V(\mathbf{r}) = \frac{1}{2} V_A \sum_{\substack{n=1 \\ \text{odd}}}^{\infty} c_k \left(\frac{r}{z_0}\right)^k P_k(\cos\theta), \tag{9}$$

with $c_1 = 0.784$ and $c_3 = 0.320$ calculated.[11] The product $c_1 c_3$ can be easily measured[1] from the resulting frequency shift

$$\frac{\Delta\omega_z}{\omega_z} = -\frac{3}{4}\left(\frac{d}{z_0}\right)^4 \frac{c_1 c_3}{(1+C_2)^2}\left(\frac{V_A}{V_0}\right)^2. \tag{10}$$

We obtain $c_1 c_3 = 0.26(1)$ which agrees with the calculated value. These values provide neither advantages nor disadvantages relative to the hyperbolic traps.

Cylindrical Penning traps not only provide a substantially simpler apparatus suited for precision measurement and mass spectroscopy, they also open the possibility of some new experiments. For example, the radiative coupling of a one-electron oscillator to modes of the cavity formed by hyperbolic trap electrodes was clearly observed,[5,15] providing the present limit to the accuracy of electron g-2 measurements.[2] The coupling is difficult to calculate for a hyperbolic cavity[16] but becomes much more tractable for a cylindrical cavity for which the mode structure is well known and the coupling can be calculated analytically.[17] Conversely, the coupling would be minimized within the cylindrical trap with open endcaps.[12] Experiments with one electron in cylindrical traps could thus lead to better understanding of the interaction with the cavity and to improved measurement of the electron and positron magnetic moments.

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